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Appendix of Proofs

(D. Hibbs and V. Piculescu, "Tax Toleration and Tax Compliance: How Geovernment Affects the Propensity of Firms to Enter the Unofficial Economy," forthcoming *American Journal of Political Science*, January 2010)

1 The Bureaucrat's Problem (Section 2)

The bureaucrat's problem is

(1)
$$\begin{aligned} &\max_{C>0} U\left(Ey_B\right) \\ \text{s.t.} \quad Ey_B &= \theta\left(S+mC\right)-\left(1-\theta\right)P \\ \theta &= e^{-\mu C}, \;\; 0<\mu<1, \quad Ey_B>0, \quad C<\infty. \end{aligned}$$

The first order condition is

$$\begin{aligned} \frac{\partial U}{\partial C} &= \frac{\partial U}{\partial Ey_B}\frac{\partial Ey_B}{\partial C}=0 \\ &\Rightarrow \frac{\partial Ey_B}{\partial C}=\frac{\partial \theta}{\partial C}\left(S+mC\right)+\theta m+\frac{\partial \theta}{\partial C}P=0. \end{aligned}$$

Since $\frac{\partial \theta}{\partial C}=-\mu e^{-\mu C}=-\mu\theta$, the first order condition implies

$$-\mu\theta\left(S+mC\right)+\theta m-\mu\theta P=0.$$

Given $\theta\neq 0$ $(C<\infty)$, $\mu m C=m-\mu\left(S+P\right)$. The optimal supply relation given in the main text follows:

(2)
$$m=\frac{\mu\left(S+P\right)}{1-\mu C}.$$

Note that $m \geq \mu(S+P)$ if and only if $0 \leq C < \dfrac{1}{\mu}$.

2 Analysis of the Firm's Optimal Decisions (Sections 2)

The firm's problem is

$$\begin{aligned}
&\max_{k,L_o,L_u,C\geq 0} \pi = (1-t)\,y_o + y_u - (1-t)\,(1+t_w)\,wL_o - wL_u - mc \\
&\text{s.t.} \quad y_o = (B)^{\delta}\left(k\overline{K}\right)^{\alpha} L_o^{\beta} \qquad (\alpha+\beta+\delta=1) \\
&\qquad y_u = (C)^{\delta}\left((1-k)\,\overline{K}\right)^{\alpha} L_u^{\beta}, \quad 0 \leq k \leq 1.
\end{aligned} \tag{3}$$

The Lagrangian function is

$$\mathcal{L} = (1-t)\,y_o + y_u - (1-t)\,(1+t_w)\,wL_o - wL_u - mc + \lambda\,(1-k)\,,$$

and the Kuhn-Tucker conditions are $k\dfrac{\partial L}{\partial k} \geq 0, L_o\dfrac{\partial L}{\partial L_o} \geq 0, L_u\dfrac{\partial L}{\partial L_u} \geq 0,$
$C\dfrac{\partial L}{\partial C} \geq 0, \lambda\dfrac{\partial L}{\partial \lambda} \geq 0.$

2.1 Optimal Decisions with Tax Evasion and Bribes

The first order conditions for an interior solution are derived from the Kuhn-Tucker conditions which are written as follows

$$k > 0, \frac{\partial L}{\partial k} = (1-t)\,\frac{\partial y_o}{\partial k} + \frac{\partial y_u}{\partial k} - \lambda = 0 \tag{4}$$

$$L_o > 0, \frac{\partial L}{\partial L_o} = (1-t)\,\frac{\partial y_o}{\partial L_o} - (1-t)\,(1+t_w)\,w = 0 \tag{5}$$

$$L_u > 0, \frac{\partial L}{\partial L_u} = \frac{\partial y_u}{\partial L_u} - w = 0 \tag{6}$$

$$C > 0, \frac{\partial L}{\partial C} = \frac{\partial y_u}{\partial C} - m = 0 \tag{7}$$

$$\lambda = 0, \frac{\partial L}{\partial \lambda} = 1 - k > 0. \tag{8}$$

The interior solution satisfies

$$0 < k < 1, L_o > 0, L_u > 0, C > 0.$$

For $k < 1$, the first order conditions above yield

$$\begin{aligned} k &= \frac{(1-t)\, y_o}{(1-t)\, y_o + y_u} \\ \beta y_o &= (1+t_w)\, w L_o, \quad \beta y_u = w L_u \\ \delta y_u &= mC. \end{aligned} \tag{9}$$

In order to calculate official output, first combine the two production functions as follows

$$\frac{y_o}{y_u} = \left(\frac{B}{C}\right)^{\delta} \left(\frac{k}{1-k}\right)^{\alpha} \left(\frac{L_o}{L_u}\right)^{\beta} (for\overline{K} \neq 0)$$

and then use (9) to write

$$\frac{y_o}{y_u} = \left(\frac{Bm}{\delta}\right)^{\delta} (1-t)^{\alpha} \left(\frac{1}{1+t_w}\right)^{\beta} (y_o)^{\alpha+\beta} \frac{1}{(y_u)^{\alpha+\beta+\delta}} \ \ (y_u \neq 0).$$

Using $\alpha + \beta + \delta = 1$, we obtain the expression for official output given in the main text:

$$y_o = \left(\frac{Bm}{\delta}\right) (1-t)^{\frac{\alpha}{\delta}} \left(\frac{1}{1+t_w}\right)^{\frac{\beta}{\delta}}. \tag{10}$$

The level of unofficial output asserted in the main text is then calculated by substituting C, k and L_u into their corresponding first order conditions (9):

$$y_u = \left(\frac{\delta y_u}{m}\right)^{\delta} \left(\frac{y_u}{(1-t)\, y_o + y_u}\right)^{\alpha} \left(\overline{K}\right)^{\alpha} \left(\frac{\beta y_u}{w}\right)^{\beta} \ (if \ y_u \neq 0).$$

We therefore obtain

$$[(1-t)\, y_o + y_u]^{\alpha} = \left(\frac{\delta}{m}\right)^{\delta} \left(\overline{K}\right)^{\alpha} \left(\frac{\beta}{w}\right)^{\beta}$$

and then solve unofficial output as a function of official output

$$y_u = \left(\frac{\delta}{m}\right)^{\frac{\delta}{\alpha}} \left(\overline{K}\right) \left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}} - (1-t)\, y_o. \tag{11}$$

By substituting y_o with (10) we arrive at the equation for y_u in the main text:

$$y_u = \left(\frac{\delta}{m}\right)^{\frac{\delta}{\alpha}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}} (1-k)\overline{K}.$$

Equations (10) and (11) are then used to derive the share of capital k as follows:

$$\begin{aligned} k &= \frac{(1-t)\, y_o}{(1-t)\, y_o + y_u} \\ &= \frac{\left(\frac{Bm}{\delta}\right)(1-t)^{\frac{\alpha+\delta}{\delta}} \left(\frac{1}{1+t_w}\right)^{\frac{\beta}{\delta}}}{\left(\frac{\delta}{m}\right)^{\frac{\delta}{\alpha}} \left(\overline{K}\right) \left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}}} \\ &= \frac{(1-t)^{\frac{\alpha+\delta}{\delta}} B \left(\frac{1}{1+t_w}\right)^{\frac{\beta}{\delta}}}{\left(\frac{\delta}{m}\right)^{\frac{\alpha+\delta}{\alpha}} \overline{K} \left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}}}. \end{aligned} \tag{12}$$

Note that unofficial output in (11) exists (i.e. $y_u > 0$) if the following condition holds:

$$\left(\frac{\delta}{m}\right)^{\frac{\delta}{\alpha}} \left(\overline{K}\right) \left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}} > (1-t)\, y_o$$

where y_o is given in (10). By substituting for y_o we obtain the condition asserted in the main text

$$t > 1 - \left(\frac{\delta}{\mu\left(S+P\right)}\right)^{\frac{\delta}{\alpha}} \left(\frac{\overline{K}}{B}\right)^{\frac{\delta}{\alpha+\delta}} \left(\frac{\beta}{w}\right)^{\frac{\beta\delta}{\alpha(\alpha+\delta)}} \left(1+t_w\right)^{\frac{\beta}{\alpha+\delta}}.$$

The right-hand side of this inequality gives the expression for the firm's toleration of taxation reported in the main text:

$$\underline{t} \equiv 1 - \left(\frac{\delta}{\mu\left(S+P\right)}\right)^{\frac{\delta}{\alpha}} \left(\frac{\overline{K}}{B}\right)^{\frac{\delta}{\alpha+\delta}} \left(\frac{\beta}{w}\right)^{\frac{\beta\delta}{\alpha(\alpha+\delta)}} \left(1+t_w\right)^{\frac{\beta}{\alpha+\delta}}. \tag{13}$$

2.2 Optimal Decisions without Tax Evasion and Bribes

If $0 \leq t \leq \underline{t}$, a corner solution is optimal which is given by the Kuhn-Tucker conditions

$$k > 0, \frac{\partial L}{\partial k} = (1-t)\frac{\partial y_o}{\partial k} + \frac{\partial y_u}{\partial k} - \lambda = 0 \tag{14}$$

$$L_o > 0, \frac{\partial L}{\partial L_o} = (1-t)\frac{\partial y_o}{\partial L_o} - (1-t)\left(1+t_w\right)w = 0 \tag{15}$$

$$L_u = 0, \frac{\partial L}{\partial L_u} = \frac{\partial y_u}{\partial L_u} - w \leq 0 \tag{16}$$

$$C = 0, \frac{\partial L}{\partial C} = \frac{\partial y_u}{\partial C} - m \leq 0 \tag{17}$$

$$\lambda > 0, \frac{\partial L}{\partial \lambda} = 1 - k = 0. \tag{18}$$

The corner solution is therefore defined by $k = 1, L_o > 0, L_u = 0, C = 0$.

If $k = 1$, official output y_o is calculated as follows. First use the first order condition (15) and write

$$y_o = (B)^{\delta}\left(\overline{K}\right)^{\alpha}\left(\frac{\beta}{w\left(1+t_w\right)}\right)^{\beta}(y_o)^{\beta}.$$

Regrouping terms, we obtain the level of official output given in the main text:

$$y_o = (B)^{\frac{\delta}{\alpha+\delta}}\left(\overline{K}\right)^{\frac{\alpha}{\alpha+\delta}}\left[\frac{\beta}{w\left(1+t_w\right)}\right]^{\frac{\beta}{\alpha+\delta}}. \tag{19}$$

2.3 Optimal Decisions with No Official Output

Assume $B = 0$ so that the firm produces no official output ($k = 0$). Then use the first order conditions for L_u and C and write the firm's level of unofficial output as

$$y_u = \left(\frac{\delta y_u}{m}\right)^{\delta}\left(\overline{K}\right)^{\alpha}\left(\frac{\beta y_u}{w}\right)^{\beta}$$

which implies

$$y_u = \left(\frac{\delta}{m}\right)^{\frac{\delta}{\alpha}}\overline{K}\left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}}. \tag{20}$$

3 Comparative Statics (Section 2)

3.1 Changes in Optimal Decisions with Tax Evasion and Bribes

In order to see how the equilibrium level of corruption changes when B, t, t_w, μ and $S+P$ change, first use (7), (10) and (11) to derive the equilibrium level of corruption:

$$C = \left(\frac{\delta}{m}\right)^{\frac{\alpha+\delta}{\alpha}}\overline{K}\left(\frac{\beta}{w}\right)^{\frac{\beta}{\alpha}}(1-k). \tag{21}$$

Next use (12) and $d\ln(1-t) = -\frac{t}{1-t}d\ln t$ to find

$$d\ln k = -\frac{\alpha+\delta}{\delta}\frac{t}{1-t}d\ln t - \frac{\beta}{\delta}d\ln\left(1+t_w\right) + \frac{\alpha+\delta}{\alpha}d\ln m + d\ln B. \tag{22}$$

Total differentiation of log Corruption (21) yields

$$
\begin{aligned}
d \ln C &= -\frac{\alpha+\delta}{\alpha} d \ln m + d \ln (1-k) \\
&= -\frac{\alpha+\delta}{\alpha} d \ln m - \frac{k}{1-k}[-\frac{\alpha+\delta}{\delta}\frac{t}{1-t} d \ln t \\
&\quad -\frac{\beta}{\delta} d \ln (1+t_w) + \frac{\alpha+\delta}{\alpha} d \ln m + d \ln B] \\
&= -\frac{1}{1-k}\frac{\alpha+\delta}{\alpha} d \ln m + \frac{\alpha+\delta}{\delta}\frac{t}{1-t}\frac{k}{1-k} d \ln t \\
&\quad + \frac{k}{1-k}\frac{\beta}{\delta} d \ln (1+t_w) - \frac{k}{1-k} d \ln B.
\end{aligned} \tag{23}
$$

To differentiate the log of the equilibrium price of corruption totally use the supply of corruption in (2) and find

$$
\begin{aligned}
d \ln m &= d \ln \mu + d \ln (S+P) - d \ln (1-\mu C) \\
&= \left(1 + \frac{\mu C}{1-\mu C}\right) d \ln \mu + d \ln (S+P) + \frac{\mu C}{1-\mu C} d \ln C \\
&= \frac{1}{1-\mu C} d \ln \mu + d \ln (S+P) + \frac{\mu C}{1-\mu C} d \ln C.
\end{aligned} \tag{24}
$$

The following system in $d \ln C$ and $d \ln m$ is obtained:

$$
\begin{bmatrix} \frac{1-k}{k} & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ -\mu C & 1-\mu C \end{bmatrix} \begin{bmatrix} d \ln C \\ d \ln m \end{bmatrix} =
$$
$$
\begin{bmatrix} -1 & \frac{t}{1-t}\frac{\alpha+\delta}{\delta} & \frac{\beta}{\delta} & 0 & 0 \\ 0 & 0 & 0 & 1 & 1-\mu C \end{bmatrix} \begin{bmatrix} d \ln B \\ d \ln t \\ d \ln (1+t_w) \\ d \ln \mu \\ d \ln (S+P) \end{bmatrix}. \tag{25}
$$

Denote the coefficient matrix of this system by $J_0 = \begin{bmatrix} \frac{1-k}{k} & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ -\mu C & 1-\mu C \end{bmatrix}$ and calculate its determinant:

$$|J_0| = \frac{1-k}{k}(1-\mu C) + \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right)\mu C = \frac{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)}{k} > 0. \tag{26}$$

Changes in the firm's activities in the official sector are obtained as follows. First, calculate the firm's demand for labor in the official sector from its first order condition:

$$\begin{aligned} L_o &= \frac{\beta y_o}{(1+t_w)\,w} \\ &= \left(\frac{Bm}{\delta}\right)(1-t)^{\frac{\alpha}{\delta}}\left(\frac{1}{1+t_w}\right)^{\frac{\beta+\delta}{\delta}}\frac{\beta}{w}. \end{aligned} \tag{27}$$

Using $d\ln(1-t) = -\frac{t}{1-t}d\ln t$, total differentiation of the log of labor employed officially (27) yields

$$d\ln L_o = d\ln B + d\ln m - \frac{\alpha}{\delta}\frac{t}{1-t}d\ln t - \frac{\beta+\delta}{\delta}d\ln(1+t_w)\,. \tag{28}$$

Total differentiation of the log of official output gives

$$d\ln y_o = \delta d\ln B + \alpha d\ln k + \beta d\ln L_o. \tag{29}$$

Similarly, for activities in the unofficial sector, first derive the firm's demand for unofficial labor from its first order condition:

$$\begin{aligned} L_u &= \frac{\beta y_u}{w} \\ &= \left(\frac{\delta}{m}\right)^{\frac{\delta}{\alpha}}\overline{K}\left(\frac{\beta}{w}\right)^{\frac{\alpha+\beta}{\alpha}}(1-k)\,. \end{aligned} \tag{30}$$

Total differentiation of the log of (30) yields the expression

$$d\ln L_u = \frac{k}{1-k}\left[\begin{array}{c} -\left(1+\frac{\delta}{\alpha}\frac{1}{k}\right)d\ln m + \frac{t}{1-t}\frac{\alpha+\delta}{\delta}d\ln t \\ -d\ln B + \frac{\beta}{\delta}d\ln\left(1+t_w\right) \end{array}\right] \tag{31}$$

and total differentiation of the log of unofficial output gives

$$d\ln y_u = \delta d\ln C - \alpha\frac{k}{1-k}d\ln k + \beta d\ln L_u. \tag{32}$$

Finally, changes in the firm's tolerance of taxation are calculated by total differentiation of the log of the tax threshold defined in (13):[1]

$$d\ln \underline{t} = \frac{1-\underline{t}}{\underline{t}}\left[\begin{array}{c} \frac{\delta}{\alpha}d\ln\mu + \frac{\delta}{\alpha}d\ln\left(S+P\right) + \frac{\delta}{\alpha+\delta}d\ln B \\ -\frac{\beta}{\alpha+\delta}d\ln\left(1+t_w\right) \end{array}\right]. \tag{33}$$

3.2 Changes in Optimal Decisions without Tax Evasion and Bribes

When the firm does not produce in the unofficial sector $k = 1$ and the firm's production is given by (19). The firm's demand for labor in this case is

$$L_o = (B)^{\frac{\delta}{\alpha+\delta}}\left(\overline{K}\right)^{\frac{\alpha}{\alpha+\delta}}\left(\frac{\beta}{w\left(1+t_w\right)}\right)^{\frac{1}{\alpha+\delta}}. \tag{34}$$

Total differentiation of the log of eq.(34) yields:

$$d\ln L_o = \frac{\delta}{\alpha+\delta}d\ln B - \frac{1}{\alpha+\delta}d\ln\left(1+t_w\right). \tag{35}$$

Therefore, total differentiation of of the log of eq.(19) for the firm's official output gives:

$$d\ln y_o = \frac{\delta}{\alpha+\delta}d\ln B - \frac{\beta}{\alpha+\delta}d\ln\left(1+t_w\right). \tag{36}$$

[1] We use $d\ln\underline{t} = -\frac{1-\underline{t}}{\underline{t}}d\ln(1-\underline{t})$.

3.3 Effects of Changes in the Profit Tax Rate t (Figure 2 in the main text)

3.3.1 Effects of Changes in t on Evading Firms

First, calculate the effects of changes in the profit tax rate on the firm's optimal output decisions when the firm is active both officially and unofficially. The effect of a change in the log of the profit tax rate on the log of the equilibrium price of corruption is $\frac{d\ln m}{d\ln t} = \frac{|J_1|}{|J_0|}$, where $J_1 = \begin{bmatrix} \frac{1-k}{k} & \frac{t}{1-t}\frac{\alpha+\delta}{\delta} \\ -\mu C & 0 \end{bmatrix}$ and $|J_1| = \mu C \frac{t}{1-t}\frac{\alpha+\delta}{\delta} > 0$. We therefore obtain

$$\frac{d\ln m}{d\ln t} = \frac{t}{1-t}\frac{\alpha+\delta}{\delta}\frac{\mu C k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{37}$$

The effect of a change in $\ln t$ on equilibrium $\ln C$ is $\frac{d\ln C}{d\ln t} = \frac{|J_2|}{|J_0|}$,

where $J_2 = \begin{bmatrix} \frac{t}{1-t}\frac{\alpha+\delta}{\delta} & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ 0 & 1-\mu C \end{bmatrix}$ and $|J_2| = (1-\mu C)\frac{t}{1-t}\frac{\alpha+\delta}{\delta} > 0$. Hence

$$\frac{d\ln C}{d\ln t} = \frac{t}{1-t}\frac{\alpha+\delta}{\delta}\frac{(1-\mu C)\,k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{38}$$

Combining (37) and (38), the net effect of a change in the log of profit taxation on the log of total bribery mC is:

$$\frac{d\ln(mC)}{d\ln t} = \frac{k(\alpha+\delta)}{(1-t)\,\delta\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]} > 0. \tag{39}$$

With respect to the firm's official activities, the effects of changes in the log of the profit tax rate on the log of capital and the log of labor deployed by the firm in the official sector are derived from (22) and (28):

$$\frac{d\ln k}{d\ln t} = -\frac{\alpha+\delta}{\delta}\frac{t}{1-t} + \frac{\alpha+\delta}{\alpha}\frac{d\ln m}{d\ln t} \tag{40}$$

$$\frac{d\ln L_o}{d\ln t} = -\frac{\alpha}{\delta}\frac{t}{1-t} + \frac{d\ln m}{d\ln t} \tag{41}$$

which using (37) simplify to

$$\frac{d\ln k}{d\ln t} = -\frac{\alpha+\delta}{\delta}\frac{t}{1-t}\frac{(1-k)\left(1+\mu C\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0 \tag{42}$$

$$\frac{d\ln L_o}{d\ln t} = -\frac{t}{1-t}\frac{(1-k)\left(\frac{\alpha}{\delta}+\mu C\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{43}$$

Substitute (42) and (43) into (29) , and derive the total effect of a change in $\ln t$ on the log of official output as

$$\frac{d\ln y_o}{d\ln t} = -\frac{t}{1-t}\frac{(1-k)\left(\frac{\alpha}{\delta}+\mu C\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{44}$$

Similarly, for firm's activity in the unofficial sector, first calculate the effect of a change in $\ln t$ on the log of labor employed unofficially

$$\begin{aligned} \frac{d\ln L_u}{d\ln t} &= \frac{k}{1-k}\frac{t}{1-t}\frac{\alpha+\delta}{\delta} - \frac{k}{1-k}\left(1+\frac{\delta}{\alpha}\frac{1}{k}\right)\frac{d\ln m}{d\ln t} \\ &= \frac{t}{1-t}\frac{\alpha+\delta}{\delta}\frac{k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0 \end{aligned} \tag{45}$$

and then substitute(38), (42) and (45) into (32) , and obtain

$$\frac{d\ln y_u}{d\ln t} = \frac{t}{1-t}\frac{\alpha+\delta}{\delta}\frac{k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{46}$$

Combining (44) and (46) , the net effect of a change in the log of the profit tax rate on the log of total output is[2]

$$(47)\qquad \begin{aligned}\frac{d\ln\left(y_o+y_u\right)}{d\ln t} &= \frac{y_o}{y_o+y_u}\frac{d\ln y_o}{d\ln t}+\frac{y_u}{y_o+y_u}\frac{d\ln y_u}{d\ln t}\\ &= -\frac{t}{1-t}\frac{k\left(1-k\right)}{1-t\left(1-k\right)}\frac{t\left(\alpha+\delta\right)-\delta\left(1-C\mu\right)}{\delta\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]}\begin{smallmatrix}<\\>\end{smallmatrix}0\\ &\quad if\ and\ only\ if\ t\begin{smallmatrix}<\\>\end{smallmatrix}\frac{\delta}{\alpha+\delta}\left(1-C\mu\right).\end{aligned}$$

The corresponding effect of a change in the log of t on the log of the share of official output in total output is

$$(48)\qquad \begin{aligned}\frac{d\ln\left(\frac{y_o}{y_o+y_u}\right)}{d\ln t} &= \frac{d\ln y_o}{d\ln t}-\frac{d\ln\left(y_o+y_u\right)}{d\ln t}\\ &= -\frac{\left(1-k\right)t\left[\alpha+k\delta+C\mu\delta\left(1-k\right)\right]}{\left[1-t\left(1-k\right)\right]\delta\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]}<0.\end{aligned}$$

3.3.2 Effects of Changes in t on Non-Evading Firms

From (36) note that a change in the profit tax rate does not affect the firm's production of official output when the firm is active only in the official sector.

3.4 Effects of Changes in the Payroll Tax Rate t_w

3.4.1 Effects of Changes in t_w on Evading Firms

The effects of changes in the of log of the payroll tax rate term $(1+t_w)$ on the logs of the equilibrium level and price of corruption are derived from (25) as follows. The effect on the log of the equilibrium price of corruption is $\frac{d\ln m}{d\ln\left(1+t_w\right)}=\frac{|J_3|}{|J_0|}$, where $J_3=\begin{bmatrix}\frac{1-k}{k} & \frac{\beta}{\delta}\\ -\mu C & 0\end{bmatrix}$ and $|J_3|=\mu C\frac{\beta}{\delta}>0$. Hence

$$(49)\qquad \frac{d\ln m}{d\ln\left(1+t_w\right)}=\frac{\mu Ck\frac{\beta}{\delta}}{\left(1-k\right)+\mu C\left(k+\frac{\delta}{\alpha}\right)}>0.$$

[2]To obtain the result we use $\frac{d\ln(x+y)}{d\ln t}=\frac{x}{x+y}\frac{d\ln x}{d\ln t}+\frac{y}{x+y}\frac{d\ln y}{d\ln t}$.

Similarly, the effect of a change in $\ln(1+t_w)$ on the log of the equilibrium level of corruption is $\frac{d\ln C}{d\ln(1+t_w)} = \frac{|J_4|}{|J_0|}$, where $J_4 = \begin{bmatrix} \frac{\beta}{\delta} & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ 0 & 1-\mu C \end{bmatrix}$ and $|J_4| = (1-\mu C)\frac{\beta}{\delta} > 0$. Therefore

$$(50) \qquad \frac{d\ln C}{d\ln(1+t_w)} = \frac{(1-\mu C)\frac{\beta}{\delta}k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

The corresponding net effect on the log of total bribery is

$$\frac{d\ln(mC)}{d\ln(1+t_w)} = \frac{k\frac{\beta}{\delta}}{1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

Using (28) and (49), the effect of a change in $\ln(1+t_w)$ on the log of labor employed by the firm in the official sector is

$$(51) \qquad \frac{d\ln L_o}{d\ln(1+t_w)} = -\frac{(1-k)\frac{\beta+\delta}{\delta}+\mu C\left(k+\frac{\beta+\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0.$$

Using (22) and (49), the effect of a change in $\ln(1+t_w)$ on the log of the fraction of the firm's capital used in the official sector is

$$(52) \qquad \frac{d\ln k}{d\ln(1+t_w)} = -\frac{\frac{\beta}{\delta}(1-k)\left(1+\mu C\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0.$$

After substitution of (51) and (52) into (29), we find that the corresponding effect on the log of official output is

$$(53) \qquad \frac{d\ln y_o}{d\ln(1+t_w)} = -\frac{\beta}{\delta}\frac{(1-k)+\mu C\frac{\delta}{\alpha}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0.$$

To derive the effects of a change in the log payroll tax rate term on the log of unofficial activities, first calculate its effects on changes in the log of labor deployed unofficially. Using (31) and (49) we obtain

$$(54)\qquad \frac{d\ln L_u}{d\ln(1+t_w)} = \frac{k\frac{\beta}{\delta}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

Next, using (32), (50), (52) and (54) find the total effect on the log of unofficial output:

$$(55)\qquad \frac{d\ln y_u}{d\ln(1+t_w)} = \frac{k\frac{\beta}{\delta}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

The net effects of changes in $\ln(1+t_w)$ on the logs of the firm's total output and the share of official output in total output are then calculated as

$$(56)\qquad \frac{d\ln(y_o+y_u)}{d\ln(1+t_w)} = -\frac{k\frac{\beta}{\delta}\left[t(1-k)+C\mu\frac{\delta}{\alpha}\right]}{[1-t(1-k)]\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]} < 0$$

$$(57)\qquad \frac{d\ln(\frac{y_o}{y_o+y_u})}{d\ln(1+t_w)} = -\frac{(1-k)(1-t)\frac{\beta}{\delta}\left(1+C\mu\frac{\delta}{\alpha}\right)}{[1-t(1-k)]\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]} < 0.$$

An increase in $\ln(1+t_w)$ has a negative effect on the log of the firm's tolerance of taxation. Using (33) we obtain

$$(58)\qquad \frac{d\ln\underline{t}}{d\ln(1+t_w)} = -\frac{1-\underline{t}}{\underline{t}}\frac{\beta}{\alpha+\delta} < 0.$$

3.4.2 Effects of Changes in t_w on Non-Evading Firms

The changes in the log of labor deployed officially and the log of official output induced by a change in $\ln(1+t_w)$ are derived from (35) and (36) as:

$$(59)\qquad \frac{d\ln L_o}{d\ln(1+t_w)} = -\frac{1}{\alpha+\delta} < 0$$

$$(60)\qquad \frac{d\ln y_o}{d\ln(1+t_w)} = -\frac{\beta}{\alpha+\delta} < 0.$$

The effect of a change in $\ln(1+t_w)$ on $\ln\underline{t}$ is the same as for evading firms.

3.5 Effects of Changes in Institutional Services B (Figure 3 in the main text)

3.5.1 Effects of Changes in B on Evading Firms

To derive the effect of a change in the log of institutional benefits on the log of the equilibrium price of corruption compute $\frac{d\ln m}{d\ln B} = \frac{|J_5|}{|J_0|}$, where

$J_5 = \begin{bmatrix} \frac{1-k}{k} & -1 \\ -\mu C & 0 \end{bmatrix}$ and $|J_5| = -\mu C$. Hence we have:

$$\frac{d\ln m}{d\ln B} = -\frac{\mu C k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{61}$$

In similar manner calculate the response of the log of the equilibrium level of corruption as $\frac{d\ln C}{d\ln B} = \frac{|J_6|}{|J_0|}$, where $J_6 = \begin{bmatrix} -1 & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ 0 & 1-\mu C \end{bmatrix}$, and $|J_6| = \mu C - 1 < 1$. The results imply

$$\frac{d\ln C}{d\ln B} = -\frac{(1-\mu C)\,k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{62}$$

The corresponding effect on the log of total bribery is

$$\frac{d\ln(mC)}{d\ln B} = \frac{d\ln m}{d\ln B} + \frac{d\ln C}{d\ln B} \tag{63}$$

$$= -\frac{k}{(1-k)+C\mu\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{64}$$

Using (22), (28) and (61), the net effects of changes in $\ln B$ on $\ln k$ and $\ln L_0$ are

$$\frac{d\ln k}{d\ln B} = \frac{(1-k)\left(\mu C\frac{\delta}{\alpha}+1\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0 \tag{65}$$

$$\frac{d\ln L_o}{d\ln B} = \frac{(1-k)+\mu C\frac{\delta}{\alpha}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{66}$$

The corresponding total net effect of a change in $\ln B$ on the log of official output is derived using (29), (65) and (66):

$$\frac{d\ln y_o}{d\ln B} = \frac{(1-k)+\mu C\frac{\delta}{\alpha}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{67}$$

In order to find the effects of a change in $\ln B$ on the firm's operations in the unofficial sector, first calculate the effect on the log of labor used unofficially implied by (31) and (61):

$$\frac{d\ln L_u}{d\ln B} = -\frac{k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{68}$$

Next calculate the response of the log of unofficial output by using (32), (62), (65) and (68):

$$\frac{d\ln y_u}{d\ln B} = -\frac{k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{69}$$

The the net response of the log of total output induced by a change in $\ln B$ is therefore

$$\frac{d\ln\left(y_o+y_u\right)}{d\ln B} = \frac{k\left[(1-k)\,t+C\mu\frac{\delta}{\alpha}\right]}{\left[1-t\,(1-k)\right]\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]} > 0. \tag{70}$$

The corresponding response of the log of the share of official output in total output is

$$\frac{d\ln\left(\frac{y_o}{y_o+y_u}\right)}{d\ln B} = \frac{(1-k)\,(1-t)\left(1+C\mu\frac{\delta}{\alpha}\right)}{\left(1-t\,(1-k)\right)\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]} > 0. \tag{71}$$

Finally, changes in $\ln B$ also have a direct positive effect on the log of the firm's tolerance to taxation given in (33) :

$$\frac{d\ln \underline{t}}{d\ln B} = \frac{1-\underline{t}}{\underline{t}}\frac{\delta}{\alpha+\delta} > 0. \tag{72}$$

3.5.2 Effects of Changes in B on Non-Evading Firms

When the firm is active only in the official sector, changes in institutional benefits do not affect the firm's use of capital, but they do affect the demand for labor and the production of output in the official sector. Using (35) and (36) we obtain

$$\frac{d\ln L_o}{d\ln B} = \frac{\delta}{\alpha+\delta} > 0 \tag{73}$$

and

$$\frac{d\ln y_o}{d\ln B} = \frac{\delta}{\alpha+\delta} > 0. \tag{74}$$

The corresponding effect on the log of the taxation toleration threshold is the same as for firms that evade taxation.

3.6 Effects of Changes in the Exposure of Bureaucratic Corruption μ (Figure 4 in the main text)

3.6.1 Effects of Changes in μ on Evading Firms

First calculate the effects of changes in $\ln\mu$ on the log of corruption unit prices. Using (25) we obtain $\frac{d\ln m}{d\ln\mu} = \frac{|J_6|}{|J_0|}$, where $J_6 = \begin{bmatrix} \frac{1-k}{k} & 0 \\ -\mu C & 1 \end{bmatrix}$, and $|J_6| = \frac{1-k}{k}$. Hence

$$\frac{d\ln m}{d\ln\mu} = \frac{1-k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{75}$$

The effect on the equilibrium level of corruption is calculated in a similar way: $\frac{d\ln C}{d\ln\mu} = \frac{|J_7|}{|J_0|}$, where $J_7 = \begin{bmatrix} 0 & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ 1 & 1-\mu C \end{bmatrix}$, and $|J_7| = -\frac{1}{k}\left(1+\frac{\delta}{\alpha}\right)$. Therefore

$$\frac{d\ln C}{d\ln\mu} = -\frac{\left(1+\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{76}$$

The corresponding net effect on total bribery is

$$\frac{d\ln(mC)}{d\ln\mu} = -\frac{k+\frac{\delta}{\alpha}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{77}$$

Using (22) and (75) we find that the effect of a change in $\ln\mu$ on the log of the share of capital deployed in the official sector is

$$\frac{d\ln k}{d\ln\mu} = \frac{(1-k)\,\frac{\alpha+\delta}{\alpha}}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{78}$$

Using (28) and (75), the corresponding effect on the log of labor employed in the official sector is

$$\frac{d\ln L_o}{d\ln\mu} = \frac{1-k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{79}$$

The combined effect of changes induced by $\ln\mu$ in $\ln L_o$ and $\ln k$ implies that the total net effect on the log of official output is

$$\frac{d\ln y_o}{d\ln\mu} = \frac{1-k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0. \tag{80}$$

A change $\ln\mu$ also affects the log of the firm's demand for unofficial labor (eqs. (31) and (75)):

$$\frac{d\ln L_u}{d\ln\mu} = -\frac{\left(k+\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{81}$$

Equations (32), (78), (76) and (81)) imply that the total net effect of a change in $\ln\mu$ on the log of unofficial output is

$$\frac{d\ln y_u}{d\ln\mu} = -\frac{\left(k+\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{82}$$

Combining (82) and (80), we see that the total net effect of achange in $\ln \mu$ on the on the log of a firm's total output is negative

$$\frac{d\ln(y_o+y_u)}{d\ln\mu}=\frac{(1-k)\left[t\left(k+\frac{\delta}{\alpha}\right)-\frac{\delta}{\alpha}\right]}{\left[1-t(1-k)\right]\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]}<0 \tag{83}$$

for any $t<\frac{\frac{\delta}{\alpha}}{k+\frac{\delta}{\alpha}}$ and $>\underline{t}$. Note that $f(t)=\frac{\frac{\delta}{\alpha}}{k+\frac{\delta}{\alpha}}$ is monotonically increasing in t from $\frac{\frac{\delta}{\alpha}}{1+\frac{\delta}{\alpha}}$ to 1 over the range $[\underline{t},1]$, and $\underline{t}$ is likely to be $<\frac{\frac{\delta}{\alpha}}{1+\frac{\delta}{\alpha}}$ for reasonable levels of corporate taxation t when $t>\underline{t}$.[3]

The effect of a change in $\ln\mu$ on the log of the official share of total output is positive and equals

$$\frac{d\ln\left(\frac{y_o}{y_o+y_u}\right)}{d\ln\mu}=\frac{(1-k)(1-t)}{1-t(1-k)}\frac{\alpha+\delta}{\alpha}\frac{1}{1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)}>0. \tag{84}$$

Changes in $\ln\mu$ also affect positively the firm's toleration of taxation. Using (33) we obtain

$$\frac{d\ln\underline{t}}{d\ln\mu}=\frac{1-\underline{t}}{\underline{t}}\frac{\delta}{\alpha}>0. \tag{85}$$

3.6.2 Effects of Changes in μ on Non-Evading Firms

When the firm does not operate in the unofficial sector, changes in μ exert no influence on the firm's level of output. However, μ does affect the firm's tolerance of taxation. The effect is the same as for evading firms.

3.7 Effects of Changes in $S+P$

3.7.1 Effects of Changes in $S+P$ on Evading Firms

Changes in the logs of bureaucratic salaries S and the penalties P affect the log equilibrium levels of both the price and the level of corruption. Concerning price, first calculate $\frac{d\ln m}{d\ln(S+P)}=\frac{|J_8|}{|J_0|}$,

[3] Note, however, that one can also have $t>\frac{\frac{\delta}{\alpha}}{k+\frac{\delta}{\alpha}}$ if $\underline{t}>\frac{\frac{\delta}{\alpha}}{1+\frac{\delta}{\alpha}}$, which might arise in settings with high levels of B, $S+P$ and (less realistically) very high levels of profit taxation $t>\underline{t}$.

where $J_8 = \begin{bmatrix} \frac{1-k}{k} & 0 \\ -\mu C & 1-\mu C \end{bmatrix}$, and $|J_8| = (1-\mu C)\,\dfrac{1-k}{k}$. We therefore obtain

$$\text{(86)} \qquad \frac{d\ln m}{d\ln(S+P)} = \frac{(1-\mu C)(1-k)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

To find the effect on the log equilibrium level of corruption compute $\dfrac{d\ln C}{d\ln(S+P)} = \dfrac{|J_9|}{|J_0|}$, where $J_9 = \begin{bmatrix} 0 & \frac{1}{k}\left(1+\frac{\delta}{\alpha}\right) \\ 1-\mu C & 1-\mu C \end{bmatrix}$, and $|J_9| = -(1-\mu C)\,\frac{1}{k}\left(1+\frac{\delta}{\alpha}\right)$. The effect therefore is

$$\text{(87)} \qquad \frac{d\ln C}{d\ln(S+P)} = -\frac{(1-\mu C)\left(1+\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0.$$

The corresponding effect on the log of total bribery is

$$\text{(88)} \qquad \frac{d\ln(mC)}{d\ln(S+P)} = -\frac{(1-\mu C)\left(k+\frac{\delta}{\alpha}\right)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0.$$

With respect to the firm's official activity, first compute effects on the logs of labor and capital. Using (22) and (86)), we obtain the effect of $\ln(S+P)$ on the log of the share of capital deployed officially as

$$\text{(89)} \qquad \frac{d\ln k}{d\ln(S+P)} = \frac{\left(1+\frac{\delta}{\alpha}\right)(1-\mu C)(1-k)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

Using (28) and (86) ,the corresponding effect on the log of labor deployed officially is

$$\text{(90)} \qquad \frac{d\ln L_o}{d\ln(S+P)} = \frac{(1-\mu C)(1-k)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

To find the total net effect of a change in $\ln(S+P)$ on the log of official output, use (29), (89) and (90), which gives

$$\text{(91)} \qquad \frac{d\ln y_o}{d\ln(S+P)} = \frac{(1-\mu C)\,1-k}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

The effect of $\ln(S+P)$ on the log of labor deployed unofficially is

$$\frac{d\ln L_u}{d\ln(S+P)} = -\frac{\left(k+\frac{\delta}{\alpha}\right)(1-\mu C)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{92}$$

Using (32), (87), (89) and (92)), the corresponding effect on the log of unofficial production is

$$\frac{d\ln y_u}{d\ln(S+P)} = -\frac{\left(k+\frac{\delta}{\alpha}\right)(1-\mu C)}{(1-k)+\mu C\left(k+\frac{\delta}{\alpha}\right)} < 0. \tag{93}$$

The total net effect of a change in $\ln(S+P)$ on the log of the firm's total output is

$$\frac{d\ln(y_o+y_u)}{d\ln(S+P)} = \frac{(1-k)\left[\left(k+\frac{\delta}{\alpha}\right)t-\frac{\delta}{\alpha}\right](1-C\mu)}{\left[1-t(1-k)\right]\left[1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)\right]} < 0, \tag{94}$$

and the corresponding effect on the log of the share of official output in total output is

$$\frac{d\ln\left(\frac{y_o}{y_o+y_u}\right)}{d\ln(S+P)} = \frac{(1-k)(1-t)}{1-t(1-k)}\frac{\left(1+\frac{\delta}{\alpha}\right)(1-C\mu)}{1-k+C\mu\left(k+\frac{\delta}{\alpha}\right)} > 0.$$

Changes in $\ln(S+P)$ affect positively the log of the firm's tax toleration threshold (eq. (33)):

$$\frac{d\ln\underline{t}}{d\ln(S+P)} = \frac{1-\underline{t}}{\underline{t}}\frac{\delta}{\alpha} > 0. \tag{95}$$

3.7.2 Effects of Changes in $S+P$ on Non-Evading Firms

When the firm does not operate in the unofficial sector, changes in S and P exert no effect on the firm's level of output, but they do affect tax toleration. The effect of $\ln(S+P)$ on $\ln\underline{t}$ is the same as for evading firms.