4

ON ANALYZING THE EFFECTS OF POLICY INTERVENTIONS: BOX-JENKINS AND BOX-TIAO VS. STRUCTURAL EQUATION MODELS*

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In recent years, interest in applying quantitative methodologies to policy-related problems has increased markedly throughout the social sciences. This chapter outlines and contrasts two approaches to estimating the effects of reforms, policy innovations, and similar discontinuous "interventions" or "treatments" on phenomena that are observed through time. The diverse range of

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substantive problems for which the intervention analysis techniques developed here are applicable include, for example: the impact of the introduction or repeal of capital punishment on murder rates; the effect of government incomes policies on wage and price inflation; and the contribution of women's suffrage, personal registration, residency laws, new ballot forms, and so on, to the secular decline in American electoral turnout since the 1890s.

The first scheme for intervention analysis treated in this chapter is based on the time-series models of Box, Jenkins, and Tiao (Box and Jenkins, 1970; Box and Tiao, 1965, 1975). This approach owes much to the conceptual work of D. T. Campbell (Campbell, 1963, 1969; Campbell and Stanley, 1966), which emphasizes that post-hoc time-series analysis can be viewed quasi-experimentally to evaluate the impact of interventions by government agencies and other institutional actors. Box-Tiao and Box-Jenkins models[1] represent time-series observations as the realization of a linear stochastic process of a autoregressive moving average or a mixed autoregressive moving average form. Hence, no attempt is made to model the causal structure generating the time-series data. Intervention occurrences are represented by binary variables (0, 1) or by related coding schemss (+1, −1, for example), and the effects of interventions (changes in the slope and/or level of the time-series) are assessed by estimating simple "transfer functions."

The second approach to intervention analysis considered here is the socalled structural equation method. Structural techniques were developed primarily in economics, but have subsequently gained wide acceptance in all of the social sciences. (See, for example, Blalock (Ed.), 1971; Goldberger, 1972; and Goldberger and Duncan (Eds.), 1973.) The principal difference between the structural equation approach and the Box-Tiao scheme is that the former involves specification and estimation of intervention effects in the context of a system of equations designed to represent the

[1] Simple versions of these models have been explicitly linked to Campbell's methodological perspective by the educational methodologists Glass, Gottman, Maguire, and Willson (Glass, 1968, 1972; Glass, Willson, and Gottman, 1972; Maguire and Glass, 1967). A number of studies by political scientists have also employed Campbell's perspective, but these analyses have relied on statistical procedures that have weak justification in the time-series context. See, for example, Caporaso and Pelowski, 1971; and Duvall and Welfling, 1973, which are conveniently collected along with related studies in Caporaso and Ross (Eds.), 1973.

causal relationships underlying the realizations of endogenous time-series. The structural approach is therefore geared to determining how and to what extent reforms or policy innovations influence endogenous phenomena as they are transmitted through a dynamic causal structure.

I discuss what appear to be lines of convergence between the two approaches in the final section.

THE BOX-TIAO (BOX-JENKINS) APPROACH

Imagine that we are dealing with a time-series of equally spaced observations on some endogenous or dependent variable Y_t, $t = 1, 2, ..., T$, and that we want to determine the impact of some exogenous treatments or policy motivated interventions. The Box-Tiao approach employs a model of the general form

$$Y_t = \sum_k \mathbf{y}_{tk} + N_t \qquad (t = 1, 2, ..., T) \tag{1}$$

where N_t denotes stochastic noise and deterministic time trends and $\sum_k \mathbf{y}_{tk}$ represents the additional effects of the interventions over noise.

Suppose that the interventions occur, say, at the nth period and thereafter, $Y_1, ..., Y_n, ..., Y_T$. The pre-intervention Y_t series is therefore driven entirely by stochastic noise and deterministic time trends. Hence:

$$\begin{aligned} N_t &= Y_t - \sum_k \mathbf{y}_{tk} \\ &= Y_t \quad \text{for } t < n \end{aligned} \tag{2}$$

Since these Y_t observations are unperturbed by external intervention, they may be analyzed to determine a time-series model for the N_t component of (1).[2] This N_t model provides a stochastic benchmark against which functions for the intervention effects ($\mathbf{y}_{tk}$) can be specified and estimated. In the Box-Tiao framework the N_t process takes the form of an autoregressive, moving average, or mixed autoregressive-moving average model of order p, d, q:

[2] In many situations the model for N_t can be developed by analyzing all Y_t observations. This would apply, for example, in cases in which a sustained intervention is believed to produce a change in the level of the series—or in cases which a one-shot nonsustained intervention produces an effect that dies out quickly. See the discussion below.

$$\Delta^d Y_t = \theta_0 + \varphi_1 \Delta^d Y_{t-1} + \cdots + \varphi_p \Delta^d Y_{t-p} - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} + a_t \tag{3}$$

where $\Delta^d Y_t$ denotes the dth backward difference of Y_t, for example,

$$\Delta Y_t \equiv Y_t - Y_{t-1}$$
$$\Delta^2 Y_t \equiv \Delta Y_t - \Delta Y_{t-1}$$

and so on; θ_0 is a constant that indexes a deterministic polynomial time trend of degree d in the Y_t;[3] φ_p and θ_q are autoregressive and moving average coefficients, respectively; and a_t is a sequence of independently distributed random variables with mean zero and variance σ_a^2.

The noise or benchmark model in (3) asserts that the dth difference of the Y_t series is generated by a linear combination of autoregressive terms and moving average shocks. Hence $\Delta^d Y_t$ depends on p lagged terms $\Delta^d Y_{t-p}$ with coefficients $(\varphi_1, \ldots, \varphi_p)$ and on a moving linear sum of q random shocks a_t with coefficients $(1, -\theta_1, \ldots, -\theta_q)$. A nonzero constant term θ_0 accomodates deterministic time trends of order d in the undifferenced Y_t; for example, if $Y_t = \mu + \theta_0 t^d +$ stochastic terms, then $\Delta^d Y_t = \theta_0 +$ stochastic terms.

Introducing the lag operator L, such that $L Y_t \equiv Y_{t-1}$ and in general $L^i Y_t \equiv Y_{t-i}$, allows the noise model in (3) to be rewritten in the form that will prove to be convenient later. Rearranging terms in (3) a bit and noticing that Δ^d may now be expressed $(1 - L)^d$, we have

$$(1 - L)^d Y_t = \theta_0 + \sum_p \varphi_p L^p (1 - L)^d Y_t + a_t - \sum_q \theta_q L^q a_t \tag{4}$$

$$Y_t (1 - L)^d (1 - \varphi_1 L - \cdots - \varphi_p L^p) = \theta_0 + (1 - \theta_1 L - \cdots - \theta_q L^d) a_t$$

$$Y_t = \frac{\theta_0 + (1 - \theta_1 L - \cdots - \theta_q L^q) a_t}{(1 - L)^d (1 - \varphi_1 L - \cdots - \varphi_p L^p)}$$

[3] Notice that this implies that $\Delta^d Y_t$ has a nonzero mean equal to $\theta_0 / 1 - \varphi_1 - \cdots - \varphi_p$.

The autoregressive-moving average (*ARMA*) model for stochastic noise, therefore, can be written as the ratio of polynomials in L. Although the representation at first appears somewhat formidable, it will be useful when developing the examples presented below.

The first task in the Box-Tiao method is to derive a model for the N_t component in (1) by fitting an appropriately specified version of (3) or (4) to the Y_t observations that are not perturbed by external interventions. As developed by Box and Jenkins (1970), this involves an iterative process of tentative model *specification*, preliminary *estimation*, a series of *diagnostic checks*, possible model respecification, and so on.

Specification

The basic noise model in (3) or (4) is fully specified by choosing the degree of differencing d, the order of the autoregressive component p, and the order of the moving average component q.

The degree of differencing d is chosen such that the differenced series is stationary and, hence, varies about a fixed mean or equilibrium level with variance independent of displacements in time and autocovariance dependent only on the magnitude of lags in time. Stationarity, therefore, means that

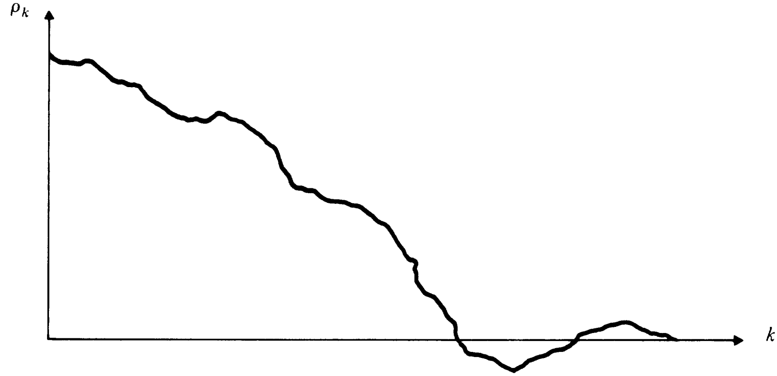
$$E(Y_t) = E(Y_{t-m})$$

and

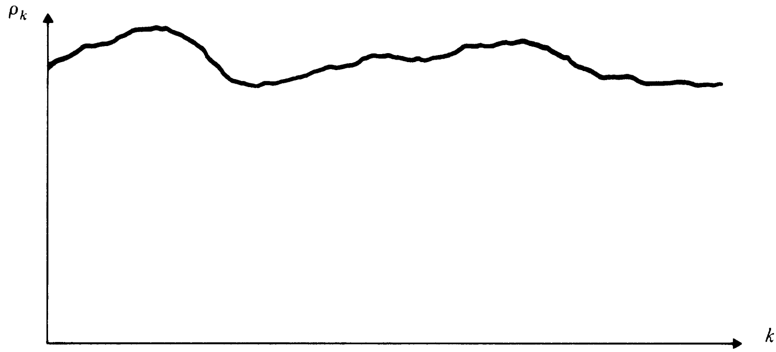
$$\mathrm{Cov}(Y_t, Y_{t-k}) = \mathrm{Cov}(Y_{t-m}, Y_{t-m-k})$$

for all t, k, and m. A stationary series will exhibit an autocorrelation function (ρ_k) that dies out after moderate-to-large lag. (A "large" lag is on the order of $k = T/5$.) Figure 1 shows hypothetical examples of the autocorrelation functions of a nonstationary and a stationary time-series. Sample estimates of the lag k autocorrelations ($\hat{\rho}_k$) are given by

$$\hat{\rho}_k = \frac{\sum_{t=k}^{T} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{T} (Y_t - \bar{Y})^2} \qquad k = 1, 2, \ldots, T/5$$



1(a) Autocorrelation Function of a Stationary Time-Series.



1(b) Autocorrelation Function of a Non-Stationary Time Series.

Figure 1. Hypothetica' Autocorrelation Function for Non-Stationary and Stationary Time-Series.

and those successive differences, $\Delta^d Y_t$, $d = 1, 2, \ldots$ are calculated analogously. In practice it is rarely necessary that d exceed 2 and, typically, $d = 1$ is sufficient to induce stationary behavior.[4]

[4] All *homogeneous* nonstationary series will exhibit stationary behavior after suitable differencing, that is, the autocorrelations of the differences $\Delta^d Y_t$ will go to zero as the lag k becomes large. Occasionally it may be necessary to apply a transformation to the Y_t in order to obtain a stationary series. For example, a series driven by an exponential function of time is *nonhomogeneous* nonstationary and, therefore, should be logarithmically transformed prior to specification and estimation.

Having settled upon a degree of differencing sufficient to ensure stationarity, the orders of the moving average and autoregressive components of (3) are tentatively specified by comparing the sample autocorrelation and partial autocorrelation functions of $\Delta^d Y_t$ to the theoretical functions of various autoregressive-moving average models. The theoretical behavior of autocorrelation and partial autocorrelation functions, denoted as ρ_k and φ_{kk}, respectively, are readily derived through algebraic manipulation of (3) for varying values of p and q. Such manipulations show that:

(1) Purely autoregressive processes of order $p[AR(p)]$ have autocorrelation functions that tail off (gradually approach zero) and partial autocorrelation functions that cut off (go to zero) after lag p. Hence ρ_k tails off, and $\varphi_{kk} = 0$ for $k > p$ in autoregressive models.

(2) Purely moving average processes of order $q[MA(q)]$ have autocorrelation functions that cut off after lag q and partial autocorrelation fumctions that tail off. Hence, $\rho_k = 0$ for $k > q$, and φ_{kk} tails off in moving average models.

(3) Mixed autoregressive-moving average processes of order $p, q[ARMA(p, q)]$ have autocorrelation functions that are a mixture of exponential and damped sine waves after the first $q - p$ lags, and partial autocorrelation functions that are dominated by a mixture of exponentials and damped sine waves after the first $p - q$ lags. Hence, neither ρ_k nor φ_{kk} cut off in mixed models.

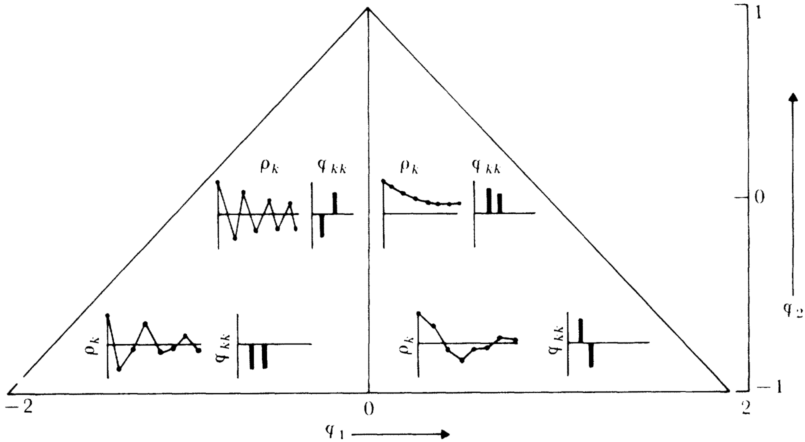
Since *AR, MA,* and *ARMA* time-series models are distinguishable by their autocorrelation and partial autocorrelation functions, sample estimates of these functions facilitate preliminary identification of p and q and permit calculation of initial guesses of the parameters φ_p and θ_q. Figure 2 and Table 1 put this into somewhat sharper focus by displaying the autocorrelation functions, partial autocorrelation functions, and related theoretical properties of some simple autoregressive moving average and mixed autoregressive-moving average models.

Estimation

The specification process outlined above leads to a tentative choice of p, d, and q, and yields preliminary guesses of the parameters φ_p and θ_q. Letting Y_t^* denote the dth difference of Y_t (that is, $Y_t^* \equiv$

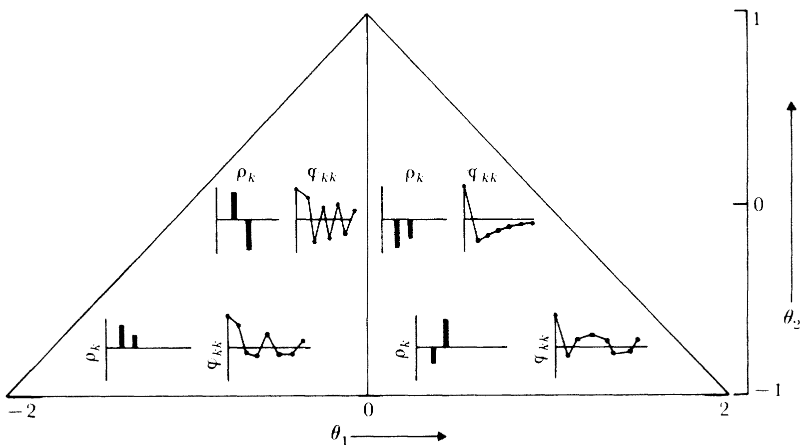
$(1 - L)^d Y_t \equiv \Delta^d Y_t)$, the autoregressive moving average model can be expressed very generally

$$Y_t^* = \theta_0 + \sum_p \varphi_p Y_{t-p}^* - \sum_q \theta_q a_{t-q} + a_t \tag{5}$$



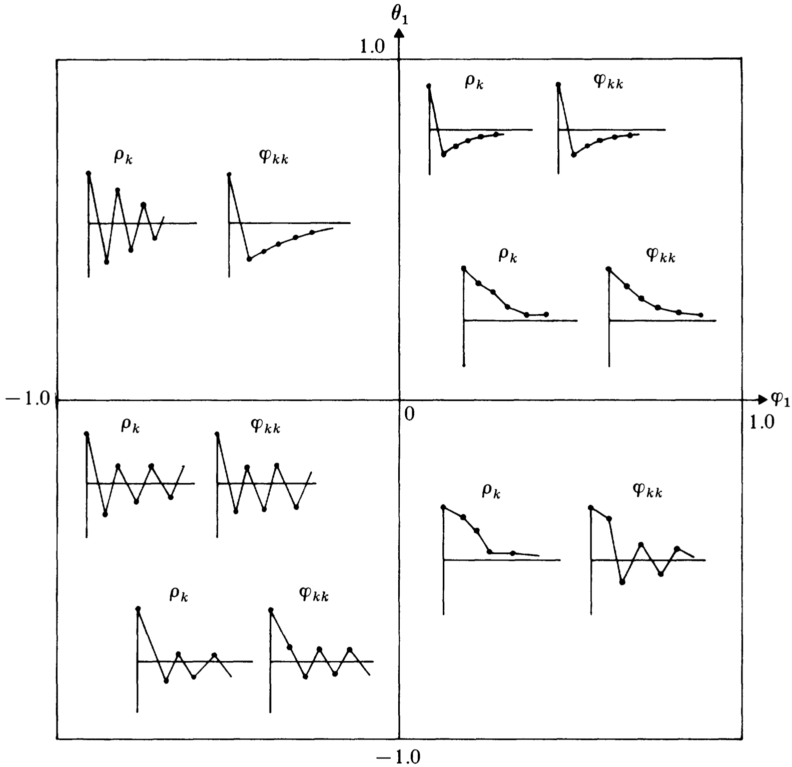
2(a) Autocorrelation and Partial Autocorrelation Functions for Various $AR(2)$ Models:

$$Y_t = \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + a_t$$



2(b) Autocorrelation and Partial Autocorrelation Functions for Various $MA(2)$ Models:

$$Y_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$



2(c) Autocorrelation and Partial Autocorrelation Functions for Various $ARMA(1,1)$ Models:

$$Y_t = \varphi_1 Y_{t-1} + a_t - \theta_1 a_{t-1}$$

Figure 2. Typical Autocorrelation (ρ_k) and Partial Autocorrelation (φ_{kk}) Functions for Various Stationary *AR, MA,* and *ARMA* Models.
(Box and Jenkins, 1970; by permission of the authors and the publisher.)

Rewriting (5) as

$$a_t = Y_t^* - \theta_0 - \sum_q \varphi_p Y_{t-p}^* + \sum_q \theta_q a_{t-q} \tag{6}$$

yields an equation for the (independently distributed) "error" term a_t. The model is estimated by choosing $(\varphi_p, \theta_q, \theta_0)$ in the admissible

TABLE 1
Some Properties of Simple *ARMA* Models of Order (p, q)
Adapted from Box and Jenkins, 1970, by permission of authors and publisher.

Order	$AR(1)$	$MA(1)$
Behavior of ρ_k	$\rho_k = \varphi_1^k$	$\rho_k = 0, \quad \text{for } k > 1$
Behavior of φ_{kk}	$\varphi_{kk} = 0, \quad \text{for } k > 1$	tails off, dominated by damped exponential
Preliminary estimates from	$\varphi_1 = \rho_1$	$\rho_1 = -\theta_1/(1 + \theta_1^2)$
Asmissible region	$-1 < \varphi_1 < 1$	$-1 < \theta_1 < 1$

Order	$AR(2)$	$MA(2)$
Behavior of ρ_k	$\rho_k = \varphi_1\rho_{k-1} + \varphi_2\rho_{k-2}$ (mixture of exponentials or damped sine wave)	$\rho_k = 0, \quad \text{for } k > 2$
Behavior of φ_{kk}	$\varphi_{kk} = 0, \quad \text{for } k > 2$	tails off, mixture of exponentials or damped sine wave
Preliminary estimates from	$\varphi_1 = \dfrac{\rho_1(1-\rho_2)}{1-\rho_1^2}$ $\varphi_2 = \dfrac{\rho_2 - \rho_1^2}{1-\rho_1^2}$	$\rho_1 = \dfrac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2}$ $\rho_2 = \dfrac{-\theta_2}{1+\theta_1^2+\theta_2^2}$
Admissible region	$-1 < \varphi_2 < 1; \varphi_2 + \varphi_1 < 1; \varphi_2 - \varphi_1 < 1$	$-1 < \theta_2 < 1; \theta_2 + \theta_1 < 1; \theta_2 - \theta_1 < 1$

Order	$ARMA(1, 1)$
Behavior of ρ_k	$\rho_k = \varphi_1^{k-1}\rho_1, \quad \text{for } k > 1$ (decays exponentially after first lag)
Behavior of φ_{kk}	$\varphi_{11} = \rho_1$, thereafter tails off dominated by damped exponential
Preliminary estimates from	$\rho_1 = \dfrac{(1-\varphi_1\theta_1)(\varphi_1-\theta_1)}{1+\theta_1^2-2\varphi_1\theta_1}$ $\rho_2 = \rho_1\varphi_1$
Admissible region	$-1 < \varphi_1 < 1; -1 < \theta_1 < 1$

parameter space[5] such that the sum of squares function

$$S(\varphi_p, \theta_q, \theta_0) = \sum_t [Y_t^* - \theta_0 - \sum_q \varphi_p Y_{t-p}^* + \sum_q \theta_p a_{t-q}]^2 \qquad (7)$$
$$= \sum_t [a_t / \varphi_p, \theta_q, \theta_0]^2$$

is minimized.

Estimates $(\hat{\varphi}_p, \hat{\theta}_q, \hat{\theta}_0)$ corresponding to a minimum of (7) are least squares estimates, and evaluating (6) at $(\hat{\varphi}_p, \hat{\theta}_q, \hat{\theta}_0)$ generates the residuals $\hat{a}_t$. (Notice that we ignore the problem of initiating the series.) In practice, the minimization of (7) may be undertaken by a number of acceptable nonlinear least-squares procedures—such as grid search, steepest descent, successive linearizations, or some combination thereof.[6] (Marquardt's (1963) compromise between the latter two methods has been popular.)

Diagnostic Checks

If the fitted model is adequate, then the calculated residuals $\hat{a}_t$ should behave as independently distributed random variates. This may be formally tested by computing the residual autocorrelations

[5] Admissibility requires that the roots of the characteristic equations

$$(1 - \varphi_1 L - \cdots - \varphi_p L^p) = 0,$$
$$(1 - \theta_1 L - \cdots - \theta_q L^q) = 0$$

(with L treated as an algebraic quantity) have roots outside the unit circle—the solutions $L_1, L_2, \ldots, L_p$ and $L_1, L_2, \ldots, L_q$ must all be greater than one in absolute value. This means that the process is stationary (if autoregressive) and invertible (if moving average) and, therefore, converges to an equilibrium level. Notice that Table 1 gives the admissible coefficient values for some simple *ARMA* models. For further discussion see Box and Jenkins, 1970; or Nelson, 1973a.

[6] Computer programs for Box-Jenkins *ARMA* model specification, estimation, and forecasting are described in Box and Jenkins, 1973, appendix (batch process programs are distributed by the Data and Program Library Service, Social Systems Research Institute, University of Wisconsin, Madison), Nelson, 1973a, appendix (batch proress programs available by writing to the author, Professor C. R. Nelson, Graduate School of Business, University of Chicago), TSP/DATATRAN manual (Cambridge Project, M.I.T., interactive computer system accessible via the ARPA national network), the TROLL Reference Manual (available from Support Staff Coordinator, NBER Computer Research Center, 575 Technology Square, Cambridge, Mass., interactive computer system accessible via the NBER's national network); and Wall, 1975 (interactive program available from the author at 575 Technology Square, Cambridge, Mass.).

$$r_k(\hat{a}) = \frac{\sum \hat{a}_t \hat{a}_{t-k}}{\sum \hat{a}_t^2}$$

and evaluating the test statistic (developed by Box and Pierce, 1970)

$$Q = (T - d) \sum_{k=1}^{K} r_k^2(\hat{a}) \qquad K \geqq 20$$

which for large K is distributed as χ^2 with $(K - p - q)$ degrees of freedom. Q serves as a general or "portmanteau" criterion of model adequacy. A large value is evidence of significant lack of fit and indicates that model respecification is necessary. Patterns in the residual autocorrelations are usually informative about the nature of the misspecification and should be analyzed along the lines proposed earlier for specification of p and q.

A well-specified *ARMA* model should, of course, also satisfy more conventional ststistical criteria of adequacy. Thus, the coefficient estimates $\hat{\varphi}_p$, $\hat{\theta}_q$, $\hat{\theta}_0$ should be significantly different from zero, and the estimated error variance $\hat{\sigma}_a^2$ should be less than that of alternative *ARMA* specifications.

Dynamic Intervention Models

The techniques outlined so far pertain to the specification, initial estimation, and diagnostic checking of the N_t component of the general Box-Tiao model in (1). As I noted earlier, the N_t process provides a stochastic benchmark against which intervention-induced changes in the slope and/or level of the endogenous Y_t series can be determined. Let us confine attention for the moment to the case of a single intervention occurring at the nth period, which is sustained thereafter. Such an intervention might be represented by the binary variable,

$$\begin{aligned} I_t &= 0 \quad \text{for } t < n \\ &= 1 \quad \text{for } t \geqq n \end{aligned}$$

Previously, the impact of an intervention on the endogenous variable, that is, the effect of I_t on Y_t, was represented simply by y_t. A general, dynamic model for the effects of exogenous interventions is given by the linear difference equation

$$y_t = \delta_1 y_{t-1} + \ldots + \delta_r y_{t-r} + \omega_0 I_{t-b} - \omega_1 I_{t-b-1} - \ldots - \omega_s I_{t-b-s} \tag{8}$$

which also can be written as the ratio of two polynomials L of degree s and r, respectively:

$$y_t(1 - \delta_1 L - \dots - \delta_r L^r) = (\omega_0 - \omega_1 L - \dots - \omega_s L^s) I_{t-b} \quad (9)$$

$$y_t = \frac{(\omega_0 - \omega_1 L - \dots - \omega_s L^s)}{(1 - \delta_1 L - \dots - \delta_r L^r)} I_{t-b}$$

where: b is a delay (lag) parameter, and the system is stable.[7] Notice that when the intervention is sustained indefinitely ($I_t = 1$ for all $t \geqq n$), the effect will eventually reach the equilibrium or steady state value.[8]

$$y^* = \frac{\omega_0 - \omega_1 - \cdots \omega_p}{1 - \delta_1 - \cdots - \delta_r} \quad (10)$$

The general intervention effects model in (8) and (9) clearly admits a wide range of possibilities, however, in most empirical work, very simple versions are likely to suffice. Figure 3 shows a few examples (from Box and Tiao, 1975). Suppose that the (sustained) intervention is believed to have produced a change in the level of the endogenous series immediately following a one period delay. The appropriate function would be

$$y_t = \omega_0 I_{t-1} \qquad \text{(Figure 3a)} \quad (11)$$

An intervention that generated a gradual change in the level of a series could be represented by the first-order dynamic model.

$$y_t = \delta_1 y_{t-1} + \omega_0 I_{t-1}$$

[7] Stability requires that the roots of the characteristic equation

$$(1 - \delta_1 L - \dots - \delta_r L^r) = 0$$

(with L treated as an algebraic quantity) lie outside the unit circle, that is, the solutions $L_1, L_2, \dots, L_r$ must all be greater than unity in absolute value, which implies that the system eventually converges to an equilibrium level. This exactly parallels the stationarity and invertibility conditions for the *ARMA* model in (3) and means that the admissible regions for the parameters are the same as those given in Table 1.

[8] Since y_t is a stable or stationary process $E(y_t) = E(y_{t-1}) = \cdots = E(y_{t-r})$ equals a constant say y^*. Taking y^* as the initial conditions of (8) gives

$$y^* - \delta_1 y^* - \cdots - \delta_r y^* = \omega_0 I_{t-b} - \cdots - \omega_s I_{t-b-s}$$

Hence, if I_t is held indefinitely at the value $+1$, then

$$y^* = \frac{\omega_0 - \omega_1 - \cdots - \omega_s}{1 - \delta_1 - \cdots - \delta_r}$$

is the equilibrium value of of y_t. Structural equation modelers will recognize this as the equilibriam multiplier which is discussed in a following section.

$$y_t(1 - \delta_1 L) = \omega_0 I_{t-1} \qquad \text{(Figure 3)}$$

$$y_t = \frac{\omega_0}{1 - \delta_1 L} I_{t-1} \tag{12}$$

$$y_t = \delta_1^t y_0 + \omega_0 \sum_{i=0}^{t-1} \delta_1^i I_{t-1-i}$$

in which the rate of adjustment to a new equilibrium depends on δ_1. A slope change intervention effect can be represented by taking δ_1 to unity, which gives

$$y_t = y_{t-1} + \omega_0 I_{t-1}$$

$$y_t(1 - L) = \omega_0 I_{t-1}$$

$$y_t = \frac{\omega_0}{1 - L} I_{t-1} \tag{13}$$

$$y_t = y_0 + \omega_0 \sum_{i=0}^{t-1} I_{t-1-i}$$

This model never adjusts to a new equilibrium level and might be used to characterize empirical situations in which convergence is very slow and occurs far beyond the period of observation.

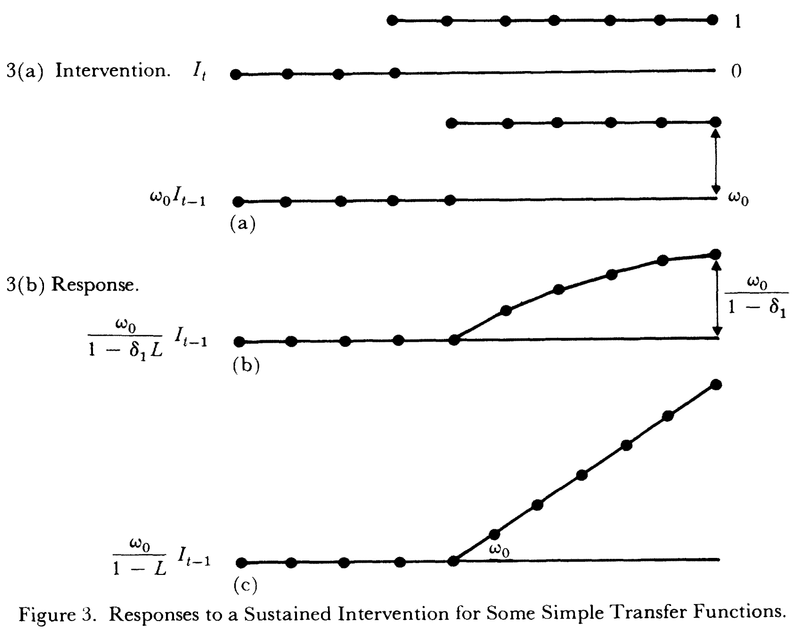


Figure 3. Responses to a Sustained Intervention for Some Simple Transfer Functions.

After a theoretically plausible intervention function has been specified (in practice, several alternatives might be entertained), it should be adjoined to the noise model whose functional form has been established by the procedures outlined previously. The parameters of the complete model can then be estimated simultaneously in order to make inferences about the impact of interventions. The scheme readily accommodates multiple interventions, a wide variety of effect patterns, and seasonal or cyclical movements in a time-series. An empirical example will undoubtedly make matters clearer.

Macroeconomic Policy and Unemployment Rate in Great Britian

An illustrative example of the Box-Tiao approach to intervention analysis is provided by my own recent study of postwar macroeconomic policy in advanced industrial societies (Hibbs, 1975). One of the central propositions in this study was that macroeconomic outcomes—especially rates of unemployment—systematically covary with the political orientation of governments. In particular, it is argued that left-wing governments assign higher priority to full employment than center- and right-wing governments; and, therefore, in *net* of trends, seasonal dependencies, and stochastic fluctuation in the unemployment time-series data, we should observe downward movement in the unemployment rate during the tenure of leftist government and upward movements in the unemployment rate during periods of centrist and rightest rule. Here we focus on the analyses for Great Britian that were designed to assess the net impact of Labour versus Conservative macroeconomic policies on the unemployment rate, as well as the effect of an important change in the British unemplotment compensation law that was initiated in 1966.

Given the general intervention analysis model $Y_t = \sum_k y_{tk} + N_t$, the first step in the model building process is to develop a preliminary specification of the stochastic *ARMA* component N_t by analyzing the sample autocorrelation functions of the endogenous unemployment variable. The sample autocorrelation function r_k for seasonally unadjusted, quarterly observations on the British unemployment rate over the 1948(1)–1972(4) period is graphed in Figure 4.[9] The sample autocorrelations decay steadily as the lag k increases,

[9]Unemployment is defined as wholly unemployed as a percentage of the civilian labor force. Since the government is always controlled by either Labour or the Conservatives, there is no "pre-intervention" time-series. Therefore, we analyze the entire unemployment series in order to develop a tentative noise model.

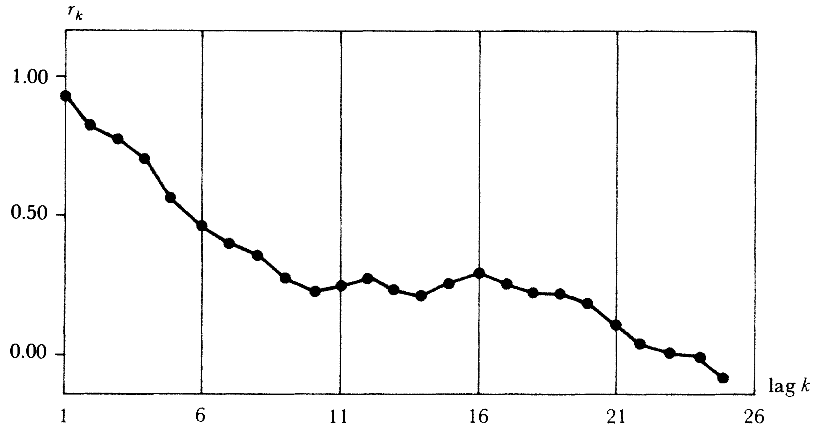


Figure 4. Sample Autocorrelation Function of British Unemployment Rate Data, 1948(1)–1972(4).

which indicates that a low order autoregressive process is compatible with the British unemployment observations (cf. Table 1 and Figure 2). Since the partial autocorrelations (which are not reported here) are insignificant for $k > 1$, we tentatively entertain a first-order autoregressive specification. Letting U_t designate the unemployment rate, we have

$$U_t = \varphi_1 U_{t-1} + e_t \tag{14}$$

or

$$(1 - \varphi_1 L) U_t = e_t$$

Figure 5 presents the sample autocorrelations of the residuals $\hat{e}_t$, that is, the autocorrelations of the transformed $U_t - \hat{\varphi}_1 U_{t-1}$. The autocorrelations exhibit distinct peaks every 4th quarter, that is, at $k = 4, 8, 12, 16, \ldots$; which suggests a strong seasonal dependence between unemployment rates of the same quarter in different years. The seasonal dependence identified in Figure 5 shows no tendency to die out as the lag k increases and, therefore, four-quarter seasonal differencing is called for. Hence we propose the model

$$(1 - L^4) e_t = \theta_0 + a_t \tag{15}$$

or

$$e_t = \frac{\theta_0 + a_t}{(1 - L^4)}$$

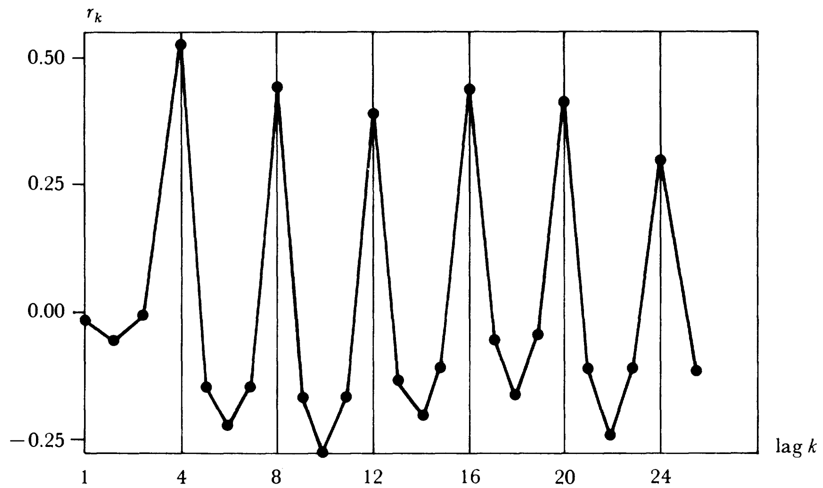


Figure 5. Sample Autocorrelation Function of the Transformed British Unemployment Rate Data $(1 - \hat{\varphi}_1 L) U_t$, 1948(1)–1972(4).

Substituting (15) into (14) yields the following expression for the stochastic noise component N_t of the general intervention analysis model:

$$(1 - \varphi_1 L) U_t = \frac{\theta_0 + a_t}{(1 - L^4)} \tag{16}$$

or

$$U_t = \frac{\theta_0 + a_t}{(1 - L^4)(1 - \varphi_1 L)} = N_t$$

Having tentatively settled on the *ARMA* specification in (15), we now propose functions for the interventions briefly described earlier. The first intervention concerns the impact of Labour versus Conservative macroeconomic policies on the postwar British unemployment rate. In view of the fact that Socialist-Labor parties typically attach much greater importance to full employment than Center or Conservative parties, we expect the unemployment rate to be driven downward during the tenure of Labour governments and to move upward during periods of Conservative rule. These effects are likely to take the form of gradual changes in the unemployment level (cf. Figure 3b) and, therefore, can be represented by the first-order dynamic expression $\mathbf{u}_{t1} = (\omega_{01}/1 - \delta_1 L)G_{t-1}$, where G_t equals $+1$ during Labour governments and -1 during Conservative governments. The intervention term G_t is specified with a one period (quarter) delay or lag, since we assume that the macroeconomic policies of a new government are not introduced or implemented instantaneously.

An important change in the British employment compensation law, which took effect in October 1966, comprises the second intervention. Until 1966 the unemployed in Great Britian received a relatively flat rate benefit that was not tied to previous earnings. The change in the unemployment system initiated in 1966 provided for an "earnings related supplement" equal to about one third of the employed person's weekly earnings, between £9 and £30. This represented a substantial increase in benefits for most wage earning groups. As a result, unemployed workers were under less financial pressure to accept unattractive jobs and presumably spent more time searching for new employment. It is therefore widely believed that the new compensation scheme increased the duration and, hence, the rate of unemployment. (See, for example, Feldstein, 1973.) Since it is reasonable to suppose that the new compensation scheme produced a *gradual* increase in the level of unemployment, we define a new variable C_t taking a value of $+1$ in 1966 (4) and thereafter, and a value of 0 otherwise; and introduce a second intervention expression $\mathbf{u}_{t2} = (\omega_{02}/1 - \delta_2 L)C_t$.

Combining the noise function proposed in (16) and the intervention expression introduced above yields the following model for the British unemployment rate

$$U_t = \mathbf{u}_{t1} + \mathbf{u}_{t2} + N_t \tag{17}$$

$$= \frac{\omega_{01}}{1 - \delta_1 L} G_{t-1} + \frac{\omega_{02}}{1 - \delta_2 L} C_t + \frac{\theta_0 + a_t}{(1 - L^4)(1 - \varphi_1 L)}$$

where:

U_t = the percentage of the civilian labor force wholly unemployed (quarterly data)

G_t = $+1$ during Labour administrations
-1 during Conservative administrations

C_t = $+1$ for 1966 (4) and thereafter, and 0 otherwise.

Equation (17) permits a simultaneous test of the hypotheses that (independent of trends, seasonal dependencies, and stochastic fluctuation in the data) the new unemployment compensation system and unrelated interparty differences in macroeconomic policy gradually altered the level of British unemployment.

The estimation results are reported in Table 2.[10] All coefficients (except the constant or trend term θ_0) are substantially larger than their estimated standard errors and, therefore, are significant by conventional statistical criteria. Before considering the implications of these estimates, let us first evaluate the adequacy of the fitted model. Figure 6 shows the actual and predicted levels of the unemployment time series.[11] The predicted unemployment observations track the actual data quite well, which of course is expected in view of the highly significant parameter estimates and small residual variance reported in Table 2. Diagnostic checks applied to the

TABLE 2
Estimation Results for the British Unemployment Rate Model (Equation 17)

	Parameter Estimates	Standard Errors
C_t	$\hat{\omega}_{01} = +0.511$	0.155
	$\hat{\delta}_1 = +0.407$	0.228
G_{t-1}	$\hat{\omega}_{02} = -0.094$	0.035
	$\hat{\delta}_2 = +0.692$	0.118
Trend (4 quarter)	$\hat{\theta}_0 = +0.002$	0.023
Autoregressive	$\hat{\varphi}_1 = +0.773$	0.071
Residual variance	$\hat{\sigma}_a^2 = 0.045$	$R^2 = 0.950$[a]

[a]The R^2 reported here pertains to the level data rather than the four-quarter difference data. The four-quarter difference R^2 is 0.850.

[10]The model was estimated with Kent D. Wall's *ERSF* program, which provides full information maximum likelihood estimates of rational distributed lag structural form equations. Details are given in Wall, 1975.

[11]The predicted level data are obtained by summing the predicted four-quarter difference series

$$\hat{U}_t = U_0 + \textstyle\sum_t (1 - L^4) U_t$$

The summation operator $\sum$ is just the inverse of difference operator $(1 - L)$ in the same way that integration is the inverse of differentiation in continuous time problems.

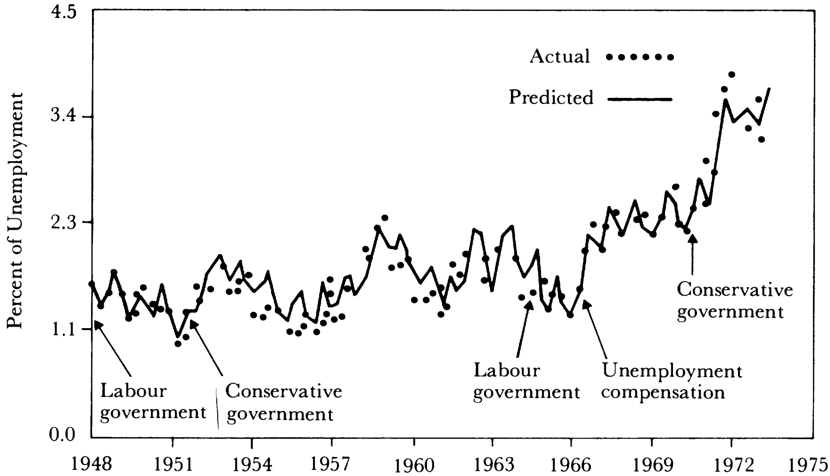


Figure 6. Actual and Predicted Values from the British Unemployment Rate Model (Equation 17).

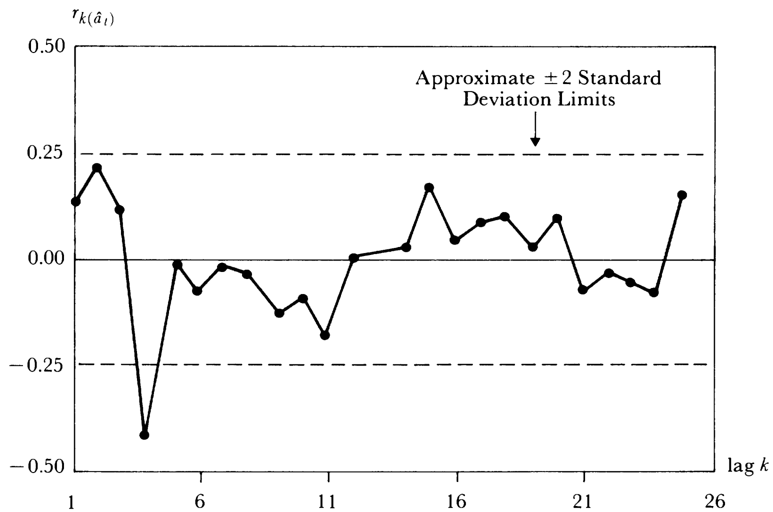


Figure 7. Residual Autocorrelations from the British Unemployment Rate Model.

residuals provide more convincing evidence of the model's adequacy. Figure 7 presents the residual autocorrelations $r_k(\hat{a}_t)$ for lags 1 through 25. The autocorrelations exhibit no systematic patterns and, except for $k = 4$, fall within the approximate ± 2 standard deviation limits.[12] The mean of the residuals is $\bar{a} = 0.0000003$ and the estimated standard error $\sigma_{\bar{a}}^2 = 0.023$. The sample evidence strongly suggests, therefore, that the a_t are independently distributed random variates with zero means.

Returning to the parameter estimates in Table 2, interest centers on the intervention coefficients $\hat{\omega}$ and $\hat{\delta}$. The coefficients associated with the unemployment compensation dummy variable $C_t(\hat{\omega}_{01}, \hat{\delta}_1)$ indicate that the additional unemployment benefits available since October 1966 produced a net increase of about 0.86 percent in the unemployment rate, that is

$$+\frac{\hat{\omega}_{01}}{1 - \hat{\delta}} = \frac{+0.511}{1 - 0.407} = 0.86$$

Holding fixed the G_{t-1} variable and the stochastic $ARMA$ terms in the model, we see that the expression

$$U_t = (\hat{\omega}_{01}/1 - \hat{\delta}_1 L)C_t$$

implies

$$U_t = \hat{\omega}_{01} \sum_{i=0}^{\infty} \hat{\delta}_1^i \; C_{t-i} \tag{18}$$

$$= \hat{\delta}_1^t U_0 + \hat{\omega}_{01} \sum_{i=0}^{t-1} \hat{\delta}_1^i C_{t-i}$$

Imposing the initial condition $U_0 = 0$ and applying the coefficient estimates $\hat{\omega}_{01} = 0.511, \hat{\delta}_1 = 0.407$, we obtain the dynamic response on the unemployment rate to the change in the unemployment compensation law by simulating (18) for C_t held at $+1$. The effect is graphed in Figure 8. In view of the fact that the dynamic response parameter $\hat{\delta}_1 = 0.407$, the steady state effect of 0.86 percent was almost fully realized rather quickly—after only 4 or 5 quarters.

[12] The lag 4 autocorrelation is, of course, significant and therefore the model might be improved by specifying $a_t = (1 - \theta_4 L^4)v_t$ where the v_t are $N(0, \sigma_v^2)$. Since the $k = 4$ autocorrelation was essentially induced by the seasonal differencing (which overcompensates for the four-quarter seasonal dependency)—and we are primarily interested in predicting the level unemployment series, modification of the model in this way is not advantageous.

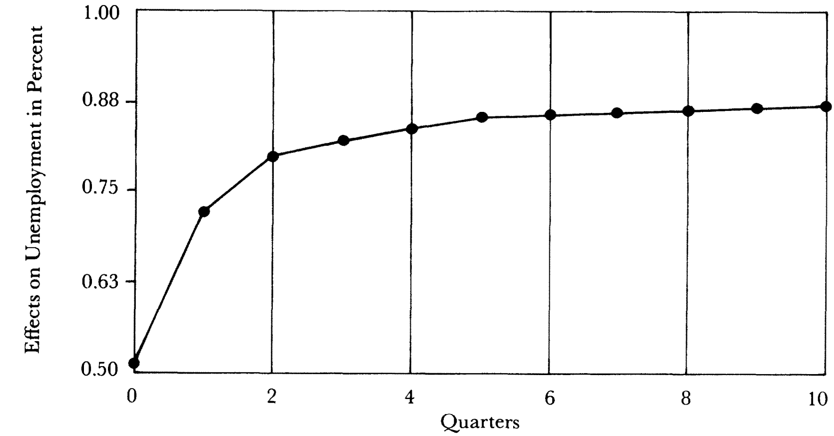


Figure 8. Simulated Net Effects of the 1966 Unemployment Compensation Law on the Unemployment Rate.

The maximum liklihood estimates of ω_{02} and δ_2 also clearly support our initial proposition concerning the impact of partisan change on the British unemployment rate. The estimates indicate that the unemployment rate is driven downward during the tenure of Labour governments and moves upward during periods of Conservative rule. The estimated steady state effects are ± 0.31 percent, that is

$$\pm \frac{\hat{\omega}_{02}}{1 - \hat{\delta}_2} = \pm \frac{0.094}{1 - 0.692} = \pm 0.31$$

which implies a difference of about 0.62 percent between the equilibrium unemployment levels associated with Labour and Conservative governments. Holding constant all other terms in the model, the expression

$$U_t = (\hat{\omega}_{02}/1 - \hat{\delta}_2 L) G_{t-1}$$

implies

$$U_t = \hat{\omega}_{02} \sum_{i=0}^{\infty} \hat{\delta}_2^i G_{t-1-i} \tag{19}$$

$$= \hat{\delta}_2^t U_0 + \hat{\omega}_{02} \sum_{i=0}^{t-1} \hat{\delta}_2^i G_{t-1-i}$$

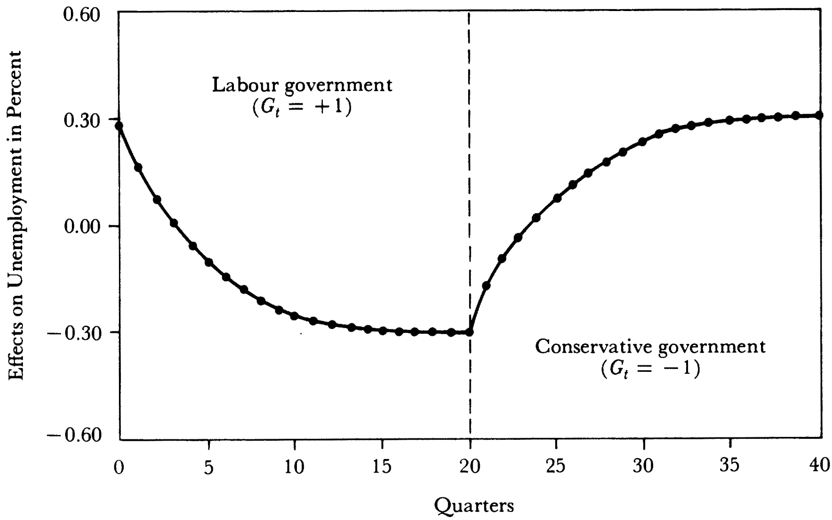


Figure 9. Simulated Net Effects of Labour and Conservative Governments on the Unemployment Rate.

Again, imposing the arbitrary initial condition $U_0 = 0$ and applying the coefficient estimates $\hat{\omega}_{02} = -0.094$ and $\hat{\delta}_2 = 0.692$, we obtain the dynamic time-paths of the unemployment rate that can be attributed to Labour and Conservative macroeconomic policies by simulating (19) for G_t held at $+1$ and -1, respectively. Figure 9 depicts the unemployment time-paths for regimes of 20 quarters (5 years) duration. Notice that the steady state of ± 0.31 percent are fully realized after about 16 quarters, or 4 years.

THE STRUCTURAL EQUATION APPROACH

In contrast to the Box-Tiao scheme, which employs a sophisticated noise model as the point of reference for assessing intervention effects, the structural equation approach attempts to represent explicitly the behavioral processes generating movements in endogenous variables. Stochastic noise in structural models is usually given little attention and is typically specified as a sequence of additive, independently distributed random variates perturbing each equation in the model.

Consider a model of m simultaneous equations ($m = 1 \ldots M$) taking the structural form

$$
\begin{aligned}
&p_{11}y_{1(t)} + \ldots + p_{1M}y_{M(t)} + \sum_i a_{11(i)}y_{1(t-i)} \\
&\quad + \ldots + \sum_i a_{1M(i)}y_{M(t-i)} + b_{11}x_{1(t)} \\
&\quad + \ldots + b_{1K}x_{K(t)} + \sum_j c_{11(j)}x_{1(t-j)} \\
&\quad + \ldots + \sum_j c_{1K(j)}x_{K(t-j)} = u_{1(t)} \\
&\qquad \vdots \\
&p_{M1}y_{1(t)} + \ldots + p_{MM}y_{M(t)} + \sum_i a_{M1(i)}y_{1(t-i)} \\
&\quad + \ldots + \sum_i a_{MM(i)}y_{M(t-i)} + b_{M1}x_{1(t)} \\
&\quad + \ldots + b_{MK}x_{K(t)} + \sum_j c_{M1(j)}x_{1(t-j)} \\
&\quad + \ldots + \sum_j c_{MK(j)}x_{K(t-j)} = u_{M(t)}
\end{aligned}
\tag{20}
$$

where: $y_{M(t)}, y_{M(t-i)}$ denote current and lagged endogenous variables, respectively; $x_{K(t)}$, $x_{K(t-j)}$ denote current and lagged exogenous variables, respectively; and $u_{M(t)}$ denotes stochastic disturbances. Without sacrificing the generality of subsequent analysis, it is convenient to confine lags in endogenous and exogenous variables to one period (any higher-order system of difference equations can be translated to a first-order system) and to write the model more compactly in matrix notation as

$$PY_t + AY_{t-1} + BX_t + CX_{t-1} = U_t \qquad (t = 1, 2, \ldots T)$$

where: P and A are $M \times M$ coefficient matrices; B and C are $M \times K$ coefficient matrices; Y_t and Y_{t-1} are M-component column vectors of current and lagged endogenous variables, respectively; X_t and X_{t-1} are K-component column vectors of current and lagged exogenous variables, respectively; and U_t is an M-component column vector of current disturbances.

If identification conditions are satisfied (see Fisher, 1966 for an exhaustive analysis), simultaneous equation models can be estimated by a variety of consistent methods; the most common being two-stage least-squares.[13] In the special case of recursive models in which P is triangular (there are no simultaneous relationships) and the cross-equation disturbance covariance matrix is diagonal (the disturbances are uncorrelated across equations), ordinary least-squares regression yields consistent parameter estimates. Throughout the discussion in this section it is assumed that the functional form of the hypothetical structural model is well established and, hence, that model validation is not an issue.[14] Attention will be confined, therefore, to techniques for intervention effects analysis in the context of a well-defined model.

Interventions and Direct Manipulation of Exogenous Variables

Intervention analysis is least problematic when the intervention or policy change is known to have been implemented by direct manipulation of exogenous variables or policy parameters. Notable examples are policy motivated, exogenously induced changes in government spending, tax rates, and the like, which figures prominently in econometric analysis of macroeconomic policy experiments.

If the structural model consists of a relatively small number of *linear* simultaneous difference equations, the response of endogenous "target" variables to exogenous interventions can be assessed analytically by the method of multiplier analysis. (See Goldberger, 1959; and Thiel and Boot, 1962.) The first step in multiplier analysis is to derive the "reduced form" of the system by solving all

[13] Note, however, that the appearance of lagged endogenous variables in (20) and (21) introduces additional complications. Briefly, consistency is not ensured unless the U_t are serially uncorrelated. If this condition fails, there are essentially two options: (1) treat the Y_{t-1} as endogenous for estimation purposes (which has obvious implications for identification); or (2) combine two-stage least-squares with generalized least squares so that the transformed disturbances are properly behaved. On the latter procedure see Fair, 1970. A general review of this and related problems is provided by Fisher, 1970a.

[14] It is hardly necessary to mention that establishing the functional form of a structural model is a substantial task, particularly in areas in which theory is not well developed and the processes under investigation are behaviorally complex. A very useful review of model evaluation procedures (which is geared to econometric systems) is given by Dhrymes *et al.*, 1972.

right-hand side current endogenous variables as functions of the predetermined lagged endogenous and exogenous variables. Thus, given the estimated structural form

$$\hat{P}Y_t = -\hat{A}\,Y_{t-1} - \hat{B}X_t - \hat{C}X_{t-1} + \hat{U}_t \tag{22}$$

the reduced form can be secured by premultiplying by $\hat{P}^{-1}$

$$Y_t = -(\hat{P}^{-1}\hat{A})\;Y_t \;-(\hat{P}^{-1}\hat{B})X_t$$

$$-(\hat{P}^{-1}\hat{C})X_{t-1} + \hat{P}^{-1}\hat{U}_t \tag{23}$$

which for convenience may be rewritten as

$$Y_t = A^*Y_{t-1} + B^*X_t + C^*X_{t-1} + V_t \tag{24}$$

where

$$A^* = -\hat{P}^{-1}\hat{A}; \qquad B^* = -\hat{P}^{-1}\hat{B};$$
$$C^* = -\hat{P}^{-1}\hat{C}; \qquad \text{and } V_t = \hat{P}^{-1}\hat{U}_t$$

Notice that every predetermined variable appears in each reduced form equation. Hence, derivation of the reduced form of the model makes explicit what is implied by the structural form; namely, that all predetermined variables directly and/or indirectly influence all endogenous variables.[15] The effects of policy motivated interventions now can be readily assessed by analyzing the reduced form in (24). Assuming that the expectation of $U_t = V_t = 0$, the immediate effects if induced changes in exogenous variables x_{kt} on the expected values of endogenous variables y_{mt} taking account of all contemporaneous feedbacks in the system, are given by elements of B^*, the so-called *impact multiplier* matrix. The elements of B^*, therefore, estimate the instantaneous impact of a unit change in x_{kt} on the *conditional* expectation of y_{mt} with the remaining exogenous variables held constant. Thus, the impact multipliers correspond to the reduced form derivatives $\partial y_{m(t)}/\partial x_{k(t)} = b^*_{mk}$. Since the model at

[15]This of course also means that the reduced form parameters can be *consistently* estimated by regressing each endogenous variable on all predetermined variables. The trade-offs between the derived reduced form estimation procedure shown in (23) and the unrestricted least-squares method mentioned here are developed in Fisher, 1965; and Goldberger, 1964, Chapter 7, Section 9.

hand is linear, endogenous responses to multiple interventions (packages of policy changes) are determined by summing over the appropriate elements of B^*; that is, by calculating

$$\sum_k \partial y_{m(t)}/\partial x_{k(t)} = \sum_k b^*_{mk}$$ [16]

Typically, interest will not be confined to the immediate consequences of policy treatments or interventions but will center instead on the dynamic, long-run implications of exogenously induced change. This amounts to investigating how the time-paths of endogenous variables are affected by external manipulation of exogenous policy instruments. Lagged, cross-temporal feedbacks in the system are now of central importance.

The effects of exogenous interventions, as they are transmitted dynamically through the model, are evaluated by lagging (24) repeatedly and substituting for lagged endogenous variables. For example, lagging (24) one period gives

$$Y_{t-1} = A^* Y_{t-2} + B^* X_{t-1} + C^* X_{t-2} + V_{t-1} \tag{25}$$

which upon substitution yields

$$Y_t = A^{*2} Y_{t-2} + B^* X_t + (C^* + A^* B^*) X_{t-1} + A^* C^* X_{t-2} + V_t + A^* V_{t-1} \tag{26}$$

Applying this procedure s times, we obtain

$$Y_t = A^{*s+1} Y_{t-s-1} + B^* X_t + \sum_{\tau=1}^{s} A^{*\tau-1}(C^* + A^* B^*) X_{t-\tau} + A^{*s} C^* X_{t-s-1} + \sum_{\tau=0}^{s} A^{*\tau} V_{t-\tau} \tag{27}$$

[16]The response to a change of any order is simply $\Delta y_{m(t)} = \sum_k b^*_{mk} \cdot \Delta x_{k(t)}$. Sociologists and political scientists will recognize that algebraic computation of the reduced form coefficients is the simultaneous equation analog of compound path analysis, which is commonly applied to static recursive models. See, for example, Stokes, 1971.

If the system is stable[17] (in which case $\lim_{s\to\infty} A^s = 0$), letting s go to infinity yields

$$Y_t = B^* X_t + \sum_{\tau=1}^{\infty} A^{*\tau-1}(C^* + A^* B^*) X_{t-\tau} + \sum_{\tau=0}^{\infty} A^{*\tau} V_{t-\tau} \tag{28}$$

which is known as the *final form* of the model.[18]

The period-by-period responses of endogenous variables to induce shifts in exogenous variables, which are known as *dynamic multipliers,* now can be obtained from (27) and (28). If the exogenous change is sustained for only one period, the effects on subsequent (expected) values of endogenous variables are given by the *delay* multiplier matrices. Hence, the estimated influence of a one-shot exogenous intervention s periods later is

[17] The stability assumption is identical to that of the previous section and essentially means that the system cannot grow or oscillate explosively without growth in exogenous variables and/or without impulses from the disturbances. Introductory accounts of the formal conditions for stability of simultaneous difference equations are given by Baumol, 1970; and Goldberg, 1958. Samuelson, 1947, provides an advanced treatment.

[18] The final form of the model can also be derived by applying the algebra of lag operators. Given the lag operator L, such that $L^i Y_t \equiv Y_{t-i}$, the reduced form of the system given in (24) can be expressed

$$(I - A^*L)Y_t = B^*X_t + C^*X_{t-1} + V_t$$

$$Y_t = (I - A^*L)^{-1} B^*X_t + (I - A^*L)^{-1} C^*X_{t-1} + (I - A^*L)^{-1} V_t$$

Since $(I - A^*L)^{-1}$ is the limit of the convergent geometric series $(I + A^*L + A^{*2}L^2 + \cdots)$ we have

$$Y_t = (I + A^*L + A^{*2}L^2 + \cdots)B^*X_t + (I + A^*L + A^{*2}L^2 + \cdots)C^*X_{t-1} + (I + A^*L + A^{*2}L^2 + \cdots)V_t$$

$$Y_t = B^*\textstyle\sum_{\tau=0}^{\infty} A^{*\tau} X_{t-\tau} + C^*\sum_{\tau=0}^{\infty} A^{*\tau} X_{t-\tau-1} + \sum_{\tau=0}^{\infty} A^{*\tau} V_{t-\tau}$$

$$Y_t = B^*X_t + \textstyle\sum_{\tau=1}^{\infty} A^{*\tau-1}(C^* + A^*B^*)X_{t-\tau} + \sum_{\tau=0}^{\infty} A^{*\tau} V_{t-\tau}$$

$$A^{*s-1}(C^* + A^*B^*) \qquad s \geqq 1 \tag{29}$$

The response of endogenous variables to one-shot exogenous impulses can, therefore, be tracked through time by evaluating the impact and successive delay multiplier matrices B^*, $(C^* + A^*B^*)$, $A^*(C^* + A^*B^*)$, $A^{*2}(C^* + A^*B^*)$, ..., the elements of which correspond to the reduced form derivatives $\partial y_{m(t+s)}/\partial x_{k(t)}$. Since the assumption of system stability implies that $\lim_{s\to\infty} A^s = 0$, it is clear that unsustained exogenous interventions will produce responses (displacements from equilibrium) in the Y_t that die out after sufficiently long lags.

Frequently, however, policy motivated interventions will be sustained through time. The dynamic implications of (unit) changes in exogenous variables that are continued, say, over s periods are given by the *cumulated multiplier* matrices

$$B^* + \sum_{\tau=1}^{s} A^{*\tau-1}(C^* + A^*B^*) \tag{30}$$

$$= B^* + (I + A^* + A^{*2} + \cdots + A^{*\tau-1})(C^* + A^*B^*)$$

which have elements corresponding to the summed reduced form derivatives

$$\sum_{\tau=0}^{s} \partial y_{m(t+\tau)}/\partial x_{k(t)}$$

The time paths of endogenous responses to sustained exogenous interventions can therefore be determined by evaluating (30) over the index τ.

Finally, by taking $\tau \to \infty$ we obtain the *equilibrium multiplier* matrices

$$\begin{aligned} B^* + \sum_{\tau=1}^{s} A^{*\tau-1}(C^* + A^*B^*) &= B^* + (I + A^* + A^{*2} + \ldots)(C^* + A^*B^*) \\ &= B^* + (I - A^*)^{-1}(C^* + A^*B^*) \qquad (31) \\ &= (I - A^*)^{-1}[(I - A^*)B^* + C^* + A^*B^*] \\ &= (I - A^*)^{-1}[B^* - A^*B^* + C^* + A^*B^*] \\ &= (I - A^*)^{-1}(B^* + C^*) \end{aligned}$$

(by the convergence rule for a geometric series).

The elements of (31) give the equilibrium or steady-state responses of endogenous variables to unit changes in exogenous variables that are sustained indefinitely.

Perhaps a simple analytic example will help clarify the results of this section. Suppose that the system under investigation is adequately represented as a pair of simultaneous equations in the structural form

$$y_{1(t)} + p_{12}y_{2(t)} + a_{11}y_{1(t-1)} + b_{11}x_{1(t)} = u_{1(t)} \tag{32}$$

$$p_{21}y_{1(t)} + y_{2(t)} + a_{22}y_{2(t-1)} + b_{22}x_{2(t)} = u_{2(t)} \tag{33}$$

In order to determine the consequences of hypothetical policy manipulations of the exogenous variables x_1 and x_2 we need to derive the reduced form of the system, which is obtained by applying simple matrix operations in the manner of (23) or, alternatively, by solving algebraically the current endogenous variables as functions of the lagged endogenous and exogenous variables. Either approach yields the reduced form equations

$$\begin{aligned} y_{1(t)} = {} & \frac{-a_{11}}{1-p_{12}p_{21}}\,y_{1(t-1)} + \frac{p_{12}a_{22}}{1-p_{12}p_{21}}\,y_{2(t-1)} \\ & - \frac{b_{11}}{1-p_{12}p_{21}}\,x_{1(t)} + \frac{p_{12}b_{22}}{1-p_{12}p_{21}}\,x_{2(t)} \\ & + \frac{1}{1-p_{12}p_{21}}\,(u_{1(t)} - p_{12}u_{2(t)}) \end{aligned} \tag{34}$$

$$\begin{aligned} y_{2(t)} = {} & \frac{p_{21}a_{11}}{1-p_{12}p_{21}}\,y_{1(t-1)} - \frac{a_{22}}{1-p_{12}p_{21}}\,y_{2(t-1)} \\ & + \frac{p_{21}b_{11}}{1-p_{12}p_{21}}\,x_{1(t)} - \frac{b_{22}}{1-p_{12}p_{21}}\,x_{2(t)} \\ & + \frac{1}{1-p_{12}p_{21}}\,(u_{2(t)} - p_{21}u_{1(t)}) \end{aligned} \tag{35}$$

In accordance with the earlier convention (c.f. equation 24) these equations are conveniently rewritten as

$$y_{1(t)} = a^*_{11}y_{1(t-1)} + a^*_{12}y_{2(t-1)} + b^*_{11}x_{1(t)} + b^*_{12}x_{2(t)} + v_{1(t)} \tag{36}$$

$$y_{2(t)} = a^*_{21}y_{1(t-1)} + a^*_{22}y_{2(t-1)} + b^*_{21}x_{1(t)} + b^*_{22}x_{2(t)} + v_{2(t)} \tag{37}$$

The instantaneous effects of unit changes in the exogenous x_{kt} on the conditional expectations of the endogenous y_{mt}, the impact multipliers, are given by the b^*_{mk}; which, as (34) and (35) make apparent, are nonlinear functions of the underlying structural coefficients in (32) and (33). The responses of the y_{mt} s periods later to interventions (which, for simplicity, are again taken to be unit changes in the x_{kt}) that are initiated at time t and sustained only 1 period are given by the delay multipliers. For example, taking $s = 2$ and applying (29) to the model at hand yields

$$\frac{\partial y_{m(t+2)}}{\partial x_{k(t)}} = A^{*2}B^* \tag{38}$$

$$= \begin{bmatrix} a^*_{11} & a^*_{12} \\ a^*_{21} & a^*_{22} \end{bmatrix}^2 \begin{bmatrix} b^*_{11} & b^*_{12} \\ b^*_{21} & b^*_{22} \end{bmatrix}$$

$$= \begin{bmatrix} (a^{*2}_{11} + a^*_{12}a^*_{21})b^*_{11} & (a^{*2}_{11} + a^*_{12}a^*_{21})b^*_{12} \\ \quad + (a^*_{11}a^*_{12} + a^*_{12}a^*_{22})b^*_{21} & \quad + (a^*_{11}a^*_{12} + a^*_{12}a^*_{22})b^*_{22} \\ (a^*_{21}a^*_{11} + a^*_{22}a^*_{21})b^*_{11} & (a^*_{21}a^*_{11} + a^*_{22}a^*_{21})b^*_{12} \\ \quad + (a^*_{21}a^*_{12} + a^{*2}_{22})b^*_{21} & \quad + (a^*_{21}a^*_{12} + a^{*2}_{22})b^*_{22} \end{bmatrix}$$

Hence, the impact of a one-shot (or pulse) unit increment in x_{1t} on $y_{1(t+2)}$ is given by the northwest entry of the matrix, that of x_{1t} on $y_{2(t+2)}$ by the southwest entry, that of x_{2t} on $y_{1(t+2)}$ by the northeast entry, and that of x_{2t} on $y_{2(t+2)}$ by the southeast entry. Delay multipliers for longer lags (leads) and/or for more complex models will obviously require even more tedious calculations, if undertaken analytically.

The cumulative effects of interventions that are sustained over some finite period are determined by application of (30), that is, by simply summing the impact and delay multiplier matrices over the appropriate time index. The ultimate impact of induced changes in exogenous variables that are maintained indefinitely are calculated by employing the equilibrium multiplier expression in (31). To illustrate for the system of (32)–(37) we derive

$$\sum_{\tau=0}^{\infty} \frac{\partial y_{m(t+\tau)}}{\partial x_{k(t)}} = (I - A^*)^{-1}B^* \tag{39}$$

$$= \begin{bmatrix} 1 - a_{11}^* & -a_{12}^* \\ -a_{21}^* & 1 - a_{22}^* \end{bmatrix}^{-1} \begin{bmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{bmatrix}$$

$$= \frac{1}{D}\begin{bmatrix} (1 - a_{22}^*)b_{11}^* + a_{12}^*b_{21}^* & (1 - a_{22}^*)b_{12}^* + a_{12}^*b_{22}^* \\ a_{21}^*b_{11}^* + (1 - a_{11}^*)b_{21}^* & a_{21}^*b_{12}^* + (1 - a_{11}^*)b_{22}^* \end{bmatrix}$$

where: $D = (1 - a_{11}^*)(1 - a_{22}^*) - a_{21}^*a_{12}^*$. The elements of (39) give the ultimate or equilibrium responses of the y_m to unit changes in the x_k that are sustained forever.

Although the analytic approach taken thus far has considerable heuristic value, the dynamic multipliers associated with changes in exogenous variables are, in practice, usually derived numerically by computer simulation. The reason, of course, is that simulated solutions are vastly more convenient computationally, even for relatively small models.[19] Simulation-based estimates of the intervention multipliers are secured by simulating the model dynamically in order to obtain an "intervention" endogenous series and a "nonintervention" endogenous series.[20] The intervention or "policy-on" solution is designed to depict actual endogenous outcomes during the post-intervention periods. Accordingly, these outcomes, which we denote as $\hat{y}_{mt}^{i}$, are generated by supplying initial conditions (values) for the y_{mt} and allowing exogenous variables and parameters to take on their historical, post-intervention values. On the other hand, the nonintervention or "policy-off" solution attempts to replicate endogenous outcomes that would have occurred

[19] Multiplier analysis in nonlinear models (that is, models in which one or more endogenous variables appear in two or more linearly independent functional forms) requires a simulation approach, since explicit analytical solutions for the reduced form equations are difficult, if not impossible, to obtain. For further discussion see Howrey and Kelejian, 1969.

[20] Dynamic simulation simply means that actual historical values of lagged endogenous variables are used only for initial conditions; all subsequent endogenous values are generated sequentially by the model. Thus, the current period's endogenous calculations form the lagged inputs to the next period, and so on.

in the absence of exogenously induced change. These outcomes, which we denote as $\hat{y}^n_{mt}$, are obtained by imposing values on exogenous variables and/or parameters that would have prevailed without the external manipulation (that is, by subtracting the known magnitude of induced changes from the historical values).

Comparison of differences $(\hat{y}^i_{mt} - \hat{y}^n_{mt})$, $(\hat{y}^i_{mt+1} - \hat{y}^n_{mt+1})$, ..., $(\hat{y}^i_{mt+\tau} - \hat{y}^n_{mt+\tau})$ yields estimates of the endogenous responses to exogenous interventions. Simulation estimates of the intervention multipliers corresponding to a sustained exogenous variable change of, say, δ—$(x_{kt} + \delta)$, ..., $(x_{kt+\tau} + \delta)$—would be given the ratios $(\hat{y}^i_{mt} - \hat{y}^n_{mt})/\delta$, ..., $(\hat{y}^i_{mt+\tau} - \hat{y}^n_{mt+\tau})/\delta$. Studies of the consequences of specific policy changes (hypothetical and actual) that have been undertaken in this way include Fromm and Taubman's (1967) analysis of the effects of the U.S. excise tax cut of 1965, and Klein's (1969) similar investigation of the U.S. income tax cut of 1964, and Klein's (1968) study of the economic consequences of Vietnam peace.

It is clear that static and dynamic multipliers are the structural equation equivalents of the Box-Tiao intervention function response schemes. However, an important advantage of the structural method is that endogenous responses to exogenous interventions can be interpreted causally in the light of structural information. That is, the behavioral mechanisms underlying intervention multipliers are made apparent by inspection of the interdependent structure of the system. Naturally, such multipliers have meaning only within the framework of the model from which they are derived. If the model does not square well with reality, then the estimated multipliers cannot be informative about real-world intervention effects.

Interventions and Structural Shifts

In most empirical situations, at least outside of macroeconomics, policy interventions are not likely to consist of direct manipulation of exogenous variables or policy parameters. On the contrary, the typical intervention will involve a change in the law, government regulation, or administrative procedure, or perhaps not an intervention in the usual sense at all—but, rather, a dramatic event such as a war, strike, critical electoral outcome, important international agreement, and so on. In such situations, the manner in which an exogenous intervention or event potentially affects a particular endogenous variable or an entire system of variables is not known a priori.

If it can be assumed that the intervention does not perturb the *values* of exogenous variables, but affects only the *parameters* of the model, the problem is readily approached by structural shift estimation. Recall that the general (m equation) linear dynamic structural model was expressed previously as

$$PY_t + AY_{t-1} + BX_t + CX_{t-1} = U_t \tag{40}$$

Rearranging terms and assuming that the system has been normalized (such that the coefficient of one endogenous variable in each structural equation is taken as $+1$ a priori) allows the equation of the model to be written in the scalar form of, for example, the jth equation:

$$\begin{aligned} y_{j(t)} = & - \sum_{m \neq j} p_{jm} y_{m(t)} - \sum_m a_{jm} y_{m(t-1)} \\ & - \sum_k b_{jk} x_{k(t)} - \sum_k C_{jk} x_{k(t-1)} + u_{j(t)} \end{aligned} \tag{41}$$

Suppose that the intervention event under investigation occurs at the nth period and continues thereafter. Shifts in the structural parameters associated with the intervention can be determined by defining a binary variable, say, D

$$\begin{aligned} D &= 0 \quad \text{for } t < n \\ &= 1 \quad \text{for } t \geqq n \end{aligned}$$

and estimating the revised, unrestricted equation(s)

$$\begin{aligned} y_{j(t)} = & - \sum_{m \neq j} P_{jm} y_{m(t)} - \sum_{m \neq j} P'_{mj} [y_{m(t)} \cdot D] \\ & - \sum_m a_{jm} y_{m(t-1)} - \sum_m a'_{jm} [y_{m(t-1)} \cdot D] \\ & - \sum_k b_{jk} x_{k(t)} - \sum_k b'_{jk} [x_{k(t)} \cdot D] \\ & - \sum_k c_{jk} x_{k(t-1)} - \sum_k c'_{jk} [x_{k(t-1)} \cdot D] + u^*_{j(t)} \end{aligned}$$

Equations in the form of (42) allow detection of structural shifts or breaks induced by the exogenous intervention by permitting all parameters to have different values in the pre- and post-intervention periods.[21] Of course, any prior (theoretical) information about the location of intervention shift effects should be exploited by setting the relevant cross-product terms equal to zero. The t ratios of p'_{jm}, a'_{jm}, b'_{jk} and c'_{jk} provide direct tests of the null hypothesis that the post-intervention parameters are not significantly different from the corresponding pre-intervention parameters. The joint hypothesis that all coefficients (or some subset thereof) are common across the pre- and post-intervention observations may be evaluated by computing the F ratio(s)[22]

$$F_{(j)} = \frac{\left[\sum_t \hat{u}^2_{j(t)} - \sum_t \hat{u}^{*2}_{j(t)}\right] \Big/ r}{\left[\sum_t \hat{u}^{*2}_{j(t)}\right] \Big/ T - G}$$

which is (are) distributed with r, and $T - G$ degrees of freedom, where: $\sum \hat{u}^2_{j(t)}$ and $\sum \hat{u}^{*2}_{j(t)}$ are estimates of the restricted and unrestricted residual sums of squares, respectively, and are derived by applying the structural coefficient estimates to the original data,[23] r denotes the number of restrictions or constraints in (41) and $T - G$ denotes the degrees of freedon of the residual sum of squares in (42)—G being the number of parameters in that equation.

Once the magnitude of parameter shifts attributable to the exogenous intervention(s) has been determined for each equation in

[21] Intercept-constants are not shown explicitly in (41) or (42), but may be considered to be among the b_{jk}. Also, there are alternative ways to set up the problem, for example, one might estimate equations in the model separately for the pre- and post-intervention periods in the spirit of analysis of covariance.

[22] If the number of parameters to be estimated exceed the available post-intervention observations, the F test of Chow, 1960, should be used in place of that given above. A unifying exposition of these and related tests is given by Fisher, 1970b.

[23] Because the the residual sums of squares are necessarily calculated in this way in simultaneous equation models, t and F statistics do not have full classical justification in the sample. They might be viewed as tests of "quasi-significance." If the model under investigation consists of a single equation that can be estimated consistently by ordinary least-squares (rather than by a simultaneous equations estimator such as two-stage least squares), $\sum_t \hat{u}^2_{j(t)}$ and $\sum_t \hat{u}^{*2}_{j(t)}$ are, of course, computed directly from the residuals of the estimating equations.

the model, the methods outlined previously for calculating intervention effects can be employed. However, the structural equation approach would appear to be of little value in situations in which the external intervention not only perturbs the model's parameters but also affects the values of exogenous variables. Unless the investigator knows which exogenous variables are affected, and by how much (a case treated earlier), there is simply no way to determine the consequences of an intervention within the structural framework. This general point is illustrated by the British unemployment example presented in the previous section. In a typical neo-Keynesian structural model, employment would depend heavily on the rate of growth of current and lagged real national income; which, in turn, would hinge on changes in government spending, taxation, investment and consumption. The impact of the supplementary unemployment benefits available since 1966 could be successfully determined in this framework by defining an intervention dummy variable (denoted earlier as C_t) and estimating revised structural eqations in the form of (42), which allow direct and indirect intervention effects. However, it is unlikely that the net effects of Labour versus Conservative macroeconomic policies on the unemployment rate could be estimated by using such techniques. The principal instruments of government macroeconomic policy are public spending and taxation, which are taken as *exogenous* in structural models. Hence, unless precise information was available about the alterations in these variables due to partisan change in the system, the structural equation approach would not permit estimation of interparty effects.

LIMITATIONS AND LINES OF CONVERGENCE

Limitations

Box-Tiao or Box-Jenkins methods are essentially models for "ignorance" that are not based on theory and, in this sense, are void of explanatory power. Although these models are in many situations likely to yield good estimates of endogenous responses to external interventions, they provide no insight into the causal structure underlying the transmission of exogenous impulses through a dynamic system of interdependent social, economic, or political relationships.

Moreover, the Box-Tiao approach is potentially susceptible to errors of inference due to "omitted variables." Discontinous movements in endogenous variables that are actually responses to discontinous changes in omitted exogenous (causal) variables are easily attributed by Box-Tiao methods to external interventions that happen to covary with sudden changes in the omitted variables. However, the multiple contrast design ("multiple-group time-series design") proposed by Campbell (1963; 1966) to deal with this problem and successfully applied in the Box-Tiao framework by Glass (1968) and Glass, Willson, and Gottman (1972) is likely to be at least somewhat effective in coping with this potential source of spurious interence.[24]

Perhaps the most obvious constraint on the use of the structural approach to intervention analysis is that many areas of inquiry, especially outside of macroeconomics, are simply not sufficiently rich in theory and/or data to permit specification and estimation of adequate structural models. In such situations the causally naive Box-Tiao scheme—which merely requires time-series observations on endogenous variables, knowledge of the time-span of external interventions, and some hunches about the form of endogenous responses—would appear to have no serious rival. However, as I noted earlier, even in areas in which acceptable structural models have been developed, the empirical data cannot be informative about intervention effects unless it can be assumed that exogenous variables do not respond to external treatments (or, at least, do not respond in ways that are not fully known a priori).

Lines of Convergence

It has been noted several times that Box-Jenkins and Box-Tiao methods are essentially sophisticated noise models that make no attempt to represent the behavioral structure generating endogenous time-series. However, recent Box-Tiao papers hint at the need to elaborate the basic "noise plus intervention function" model

[24] A good example of this design is provided by Glass' 1968 study of the effectiveness of Governor Abraham Rubicoff's 1955 "crackdown" on speeding in reducing traffic fatalities in Connecticut. In order to ensure that the effects attributed to this intervention were genuine, Glass analyzed the fatality rates of four "control" states that did not experience a comparable alteration of law enforcement practices.

to incorporate additional exogenous variables and interdependent relationships among "output" or endogenous variables. Progress along these lines has already been made by Wall (1974) and Wall and Westcott (1974) and this work clearly points in the structural equation direction.

Conversely, the structural equation tradition has placed great emphasis on behavioral sophistication but has given much less attention to noise or disturbance processes. Error models other than first-order autoregressive schemes are rarely entertained in empirical studies; indeed, in simultaneous equation models the disturbances are nearly always assumed at the outset to be white noise. This may, in part, underlie the rather poor short-term forecasting performance of econometric models in relation to that of naive, especially Box-Jenkins, alternatives. (Cf. Cooper, 1972; Naylor *et al.,* 1972; Nelson, 1973; Stekler, 1968).

However, structural modelers are becoming more sensitive to the need for stochastic sophistication. A number of recent state of the art papers have urged that greater attention be given to error processes (Dhrymes, *et al.,* 1972; Klein, 1971), and work on the specification and estimation of more complex disturbance models in the structural context is beginning to appear with regularity in the technical literature. (See, for example, Chow and Fair, 1973; Fair, 1970, 1973; Hannan and Nichols, 1972; Hibbs, 1974; Sarris and Eisner, 1974; and Schmidt, 1971.) As I have tried to show in an earlier paper (Hibbs, 1974), Box-Jenkins techniques are ideally suited to the characterization of structural disturbance processes, which, after all, represent our ignorance. Finally, the traditional econometric commitment to the maintained hypothesis and strong axiomatization of models appears to be giving way to a renewed emphasis on experimentation with functional forms. These developments in the structural equation camp have much in common with the explicit empiricism of the *ARMA* approach and clearly point in the Box-Jenkins direction. Indeed, Box-Jenkins techniques applied to the final autoregressive equations of econometric models have been shown to be useful in validating the adequacy of the presumed causal structure. (See Pierce and Mason, *n.d.,* and Zellner and Palm, 1973.)

Convergence will be fully realized when structural models corrupted by *ARMA* noise are used routinely in empirical work.

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