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Aggregate Supply, Output Fluctuations and Policy Neutrality

In this lecture I'll illustrate how several specific routes by which nominal misperceptions may cause output fluctuations can be viewed as variations on the basic setup of the famous 'Lucas supply function'. Along the way I'll give a brief history of the Phillips curve, and I will show how it connects to, and ultimately led to, the subsequent textbook Lucas supply equation. Then we'll look at the implications of Lucasian supply functions for monetary policy "neutrality" – an implication of Lucas' seminal work that was quickly sharpened by Barro, Sargent and Wallace, and comprised a major assault on traditional Keynesian models which featured sluggish adjustment of nominal wages and prices as a key source of business cycles. Subsequent lectures will build upon these foundations.

Unless noted otherwise, all lower case variables denote corresponding upper case variables in natural log form.

1 Nominal Adjustment Theories of Business Cycles

Nominal adjustment theories of the business cycle operate through an aggregate supply equation. The underlying imperfections in information about prices and/or the money supply could logically originate with the process of wage formation in the labor market, or with production decisions in the goods market, or both.

The benchmark equation is the supply function found in most intermediate macroeconomics textbooks (with random disturbances and other exogenous influences omitted):

$$q_t^S = q_t^* + \beta \left[p_t - E\left(p_t | I_t\right)\right], \qquad \beta > 0 \tag{1}$$

where q^S is log aggregate supply, q^* is log "potential", "normal" or "natural" output – the log output level that would be ground out by a competitive economy in equilibrium,[1] p is log output price, and $E(p_t|I_t)$ is the expectation of price conditioned on the time t information set of agents (which I shall sometimes write as $E_{t-1}X$), where it is assumed that I_t is restricted to outcomes realized at $t-1$ and earlier.[2] The natural log output level, q_t^*, is known to all agents, and deviations of log output supply from q^* are driven by price (or inflation)[3] "surprises". The same asserted supply model could readily be obtained from a simple wage-formation setup.

1.1 *A Labor Market View*

Using eq.(1) as the benchmark, in the labor market view we have

$$q_t^S = q_t^* + \beta \cdot (p_t - \hat{w}). \tag{2}$$

Output deviates from potential in proportion β to movements in the productivity-adjusted real wage. The log nominal wage (net of productivity)[4], $\hat{w}$, is determined by expected log price

$$\hat{w} = E_{t-1}p_t, \tag{3}$$

[1] Often q^* is modelled as a linear trend (as Lucas did in several papers), which would be consistent with what we used to obtain stylized results in the growth theory lectures. Hence on the balanced growth path with labor growing at rate n and technology at rate g,output grows at rate

$$\Delta q^* = (g+n),$$

and log natural output is therefore

$$q_t^* = q_0 + (g+n) \cdot t.$$

[2] In earlier lectures it was often posited that the time t information set included outcomes realized up to and *including* time t. Keep the new assumption in mind.

[3] Deviating the price terms in (1) gives the same model with inflation surprises in place of log price surprises.

[4] Recall that in a competitive setting with technology growing exponentially – $A_t = A_0(1+g)^t$ – the real wage can be expressed in discrete time as

$$\begin{aligned}\frac{W_t}{P_t} &= A_t \cdot \left[\frac{Q_t}{A_t \cdot N_t} - F'\left(\frac{K_t}{A_t \cdot N_t}\right) \cdot \frac{K_t}{A_t \cdot N_t}\right] \\ &= \frac{\partial Q_t}{\partial N_t}.\end{aligned}$$

$\hat{w}_t$ can be thought of as $\log\left(\frac{W_t}{A_t}\right)$.

which gets us back to eq.(1).

As a simple illustrative motivation of the labor market view, consider a production technology in which labor is the only endogenous input:

$$Q_t^S = A_t N_t^{\alpha}. \tag{4}$$

Let log labor demand, n^D, be obtained from the FOC for static profit maximization:

$$Q'(N_t) = \alpha A_t N_t^{\alpha-1} = \frac{W_t}{P_t}, \tag{5}$$

which gives log labor demand as

$$n_t^D = \frac{1}{(1-\alpha)} \cdot [\ln \alpha + (p_t - \hat{w})], \qquad \hat{w} \equiv [\hat{w} - \ln A_t]. \tag{6}$$

Using these simple relations we arrive at the log aggregate supply function

$$q_t^S = q_t^* + \frac{\alpha}{(1-\alpha)} \cdot (p_t - \hat{w}), \tag{7}$$

which is eq.(2) with $q_t^* = \left[\frac{\alpha}{(1-\alpha)} \ln \alpha + \ln A_t\right]$ and $\beta = \frac{\alpha}{(1-\alpha)}$. Output therefore travels up the production function on the back of falling real wages. (Whence the knee-jerk reaction from economists to output and employment problems: "cut wages".) When wages are formed according to

$$\hat{w}_t = E_{t-1} p_t \tag{8}$$

we arrive at the benchmark supply function in (1).

1.2 *An Historical Diversion on the Phillips Curve*

A precursor of the modern aggregate supply function was the famous Phillips curve (after A.W. Phillips, Economica, 1958), which based on historical evidence identified a convex relation between wage inflation or price inflation and the rate of unemployment

$$\hat{w} - w_{t-1} = a - b u_t \tag{9}$$

$$p_t - p_{t-1} = a - b u_t \tag{10}$$

u being the log of the unemployment rate, U. Money wage inflation and unemployment were regarded by many as bearing a "trade-off' relationship – a view that two giants of economics – Paul Samuelson and Robert Solow – argued (with subtle reservations) provided policy authorities with something like a "menu of choices" for aggregate demand policy. (Sameulson and Solow, AER, 1960).

Almost simultaneously (a common occurrence in science) Milton Friedman (AER, 1968) and Edmund Phelps (Economica, 1967) pointed out that a real variable like unemployment could only be related to a nominal variable like the wage inflation rate when the inflation expectations affecting wage formation lagged behind actual inflation developments.[5] This view yielded 'expectations augmented' Phillips curve equations like

$$w_t - w_{t-1} = a - b\left(U_t - U^*_{(t)}\right) + c \cdot E_{t-1}\left(p_t - p_{t-1}\right) \tag{11}$$

and

$$p_t - p_{t-1} = a - b\left(U_t - U^*_{(t)}\right) + c \cdot E_{t-1}\left(p_t - p_{t-1}\right), \tag{12}$$

where $U^*_{(t)}$, is the so-called 'natural rate of unemployment,' which you can take to be the unemployment rate when output is at equilibrium, q^*_t.

Expected inflation was commonly modelled adaptively (cf. the Consumption lecture notes on Friedman's adaptive expectations proposal for modelling permanent income), and many studies attempted to determine whether or not the coefficient c was less than 1.0 or not, which was considered informative about the slope of the short- and long-run Phillips curve and, indeed, whether a long-run Phillips curve "trade-off" existed at all.[6]

All this has gone by the boards under more modern views. No one seriously entertains the idea anymore that there could possibly be a stable "menu of choices" allowing policy makers to use aggregate demand policy to select combinations of unemployment and inflation, as traced out by a stable, downward sloping Phillips curve schedule in inflation-unemployment space.

By Okun's law (Arthur Okun,1962 ASA, and taken from the start to be a statistical rather than a structural relation) we know that unemployment and output generally bear close association to one another

$$\left(q_t - q^*_t\right) = k - \theta\left(U_t - U^*_{(t)}\right) \tag{13}$$

[5]The idea that expectations enter the story, however, can be traced back further; at least to Ludwig von Mises (The Theory of Money and Credit, 1953).

[6]Thomas Sargent made one of his early impacts on the economics profession (JPE, 1976) by pointing out that the magnitude of "c" was irrelevant to the trade-off view when expectations were modelled adaptively, or modelled in any other fashion that did not correspond to a rational model of expectation formation in Muth's (Econometrica, 1961) sense.

where in US data θ is in the vicinity of 0.02 to 0.025.[7] By the 1970s what was once studied in Phillips curve fashion was more commonly written as an aggregate supply relation, which amounts to interchanging inflation and unemployment on the left- and right-sides of eq.(12), and using Okun's law to substitute out $\left(U_t - U^*_{(t)}\right)$:

$$\left(U_t - U^*_{(t)}\right) = \frac{a}{b} - \frac{1}{b}\left(p_t - p_{t-1}\right) + \frac{c}{b} \cdot E_{t-1}\left(p_t - p_{t-1}\right) \tag{14}$$

$$\left(q_t - q^*_t\right) = k - \theta\frac{a}{b} + \theta\frac{1}{b}\left(p_t - p_{t-1}\right) - \theta\frac{c}{b} \cdot E_{t-1}\left(p_t - p_{t-1}\right). \tag{15}$$

At $c = 1$ (presumed under rationality of expectations)

$$q_t = k - \theta\frac{a}{b} + q^*_t + \theta\frac{1}{b}\left[\left(p_t - p_{t-1}\right) - E_{t-1}\left(p_t - p_{t-1}\right)\right], \tag{16}$$

which up to a scalar is observationally equivalent to the supply schedule in eq.(1).

2 The Lucas Supply Model

The benchmark aggregate supply function on offer in macroeconomics originates with Lucas (JET, 1972 ; AER,1973 and Lucas and Rapping, JPE, 1969). The mechanics and logical foundation laid out in the 1973 paper are especially famous, and you should be thoroughly familiar with them. The main (and quite preposterous) idea – said to taken as a "metaphor" rather than as a plausible description of any real world economy – is that producers cannot distinguish the relative price movements they want to respond to from general price movements (inflation). They rationally attempt to make the distinction, but there is of course temporary slippage (inherently transitory "surprises") which gives the statistical appearance of a upward sloping aggregate supply curve. Observing an upward sloping supply curve in data, policy-makers consequently might erroneously try to exploit it by jacking up aggregate demand in the hopes of increasing real incomes. [*Graph*]

[7] In other words, measured on an annual basis output is depressed around 2.0 to 2.5 percentage points for each extra percentage point of unemployment. The magnitude of this statistical parameter is clearly contingent on the institutional setting. It tends to be much bigger in many postwar European macroeconomies than in the US. Think about the relationship the other way around, and ask yourself why might unemployment tend to fall less during booms and rise less during busts in Europe than in America? And if this be true, why might the natural rate of unemployment (structural unemployment) be higher in the latter setting.

2.1 *The State of the World*

- Goods are traded in physically isolated competitive markets ("islands"). Demand is distributed unevenly across the markets, which yields relative as well as general price movements. Producers have perfect, instantaneous information about own-price in their own market, but imperfect information about the prices producers face in other markets and, hence, imperfect information about the general price level.
- Aggregate Demand is 'Keynesian' (IS-LM), and so it can be shifted by movements in M, G, T and X. To keep simple the partitioning of nominal output, Y, into real quantities, Q, and prices, P, it is assumed that demand is unit elastic. This allows us to plug into the quantity identity

$$Y \equiv P \cdot Q = M \cdot V \tag{17}$$

$$V = \frac{Y}{M} = 1.0 \tag{18}$$

so that nominal spending equals the money supply[8]

$$Y = M, \tag{19}$$

and we can write the log aggregate demand equation

$$q_t^D = m_t - p_t. \tag{20}$$

2.2 *Demand in Market* i

Demand for the representative good in market i is

$$Q_i^D = Q \left(\frac{P_i}{P} \right)^{-\eta} \varepsilon_i^D, \qquad \epsilon^D \sim \log \text{Normal} \tag{21}$$

where the setup imposes the (inessential) simplifying assumption that $\eta = 1$, so that aggregate price is just the geometric mean

$$P = \Pi_{i=1}^J P_i^{\frac{1}{J}} \tag{22}$$

[8] M might be thought of quite broadly as any variable affecting aggregate demand, rather than as the money supply per se. The important point is that aggregate demand is unit elastic. Setting velocity to 1.0 is just done for convenience, without loss of generality. We could carry aggregate demand velocity shocks throughout the story.

and log aggregate price is

$$p = \frac{1}{J}\sum_{i=1}^{J} p_i = \overline{p_i}. \tag{23}$$

Market-specific log demand is therefore

$$q_i^D = q - (p_i - p) + e_i^D, \qquad e^D \equiv \ln \epsilon^D \sim 0,\ \sigma_{e^D}^2 \tag{24}$$

2.3 *Supply in Market* i

To motivate simply the Lucas supply function from highly simplified first principles, imagine that production possibilities for the representative good are

$$Q_i^S = A_{(t)} N_i^{\alpha} \varepsilon_i^S, \qquad \alpha \in (0,1)\,,\ \varepsilon^S \sim \log \text{Normal} \tag{25}$$

where N is producers' own labor, A is the state of technology and we abstract from other inputs to production.

Producer utility is given by

$$U_i = \frac{P_i \hat{w}_i}{P} - \frac{1}{\gamma} N_i^{\gamma}, \qquad \gamma > \alpha \tag{26}$$

where $\hat{w}_i = \frac{P_i Q_i}{P} \cdot \frac{1}{A_{(t)}}$ is real income from production net of the secular rise in productivity. U_i implies that producers have constant, unit marginal utility of net income. Substituting for income, utility is

$$U_i = \frac{P_i N_i^{\alpha} \varepsilon_i^S}{P} - \frac{1}{\gamma} N_i^{\gamma}. \tag{27}$$

P and P_i (aggregate price and local market price) are exogenous to local producers, so having substituted out $\hat{w}_i$ we have but one FOC:

$$\frac{\partial U_i}{\partial N_i} = \alpha \frac{P_i N_i^{\alpha-1} \varepsilon_i^S}{P} - N_i^{\gamma-1} = 0, \tag{28}$$

$$\Rightarrow \alpha \frac{P_i \varepsilon_i^S}{P} = N^{(\gamma-\alpha)}.$$

Taking logs and finding n_i we obtain[9]

[9]Note that n now grows indefintiely with A!

$$n_i = \frac{1}{(\gamma - \alpha)} \cdot \left[(p_i - p) + e_i^S\right], \quad e^S \equiv \ln \varepsilon^S \sim 0,\ \sigma^2_{e^S} \tag{29}$$

The log supply function in market i is found by taking the log of eq.(25) and using (29) to substitute for n_i :

$$q_i^S = \ln A + \frac{\alpha}{(\gamma - \alpha)} (p_i - p) + \frac{\gamma}{(\gamma - \alpha)} e_i^S. \tag{30}$$

2.4 *Equilibrium Price*

We now find equilibrium price. As usual, equate supply and demand and solve for p_i

$$q_i^D = q_i^S \tag{31}$$

$$\left\{ \begin{array}{c} \left[q - (p_i - p) + e_i^D\right] = \\ \\ \left[\ln A + \frac{\alpha}{(\gamma - \alpha)} (p_i - p) + \frac{\gamma}{(\gamma - \alpha)} e_i^S\right] \end{array} \right\}, \tag{32}$$

which after a bit of boring algebra gives p_i as

$$p_i = p - \frac{(\gamma - \alpha)}{\gamma} \ln A + \frac{(\gamma - \alpha)}{\gamma} q + \frac{(\gamma - \alpha)}{\gamma} e_i^D - e_i^S. \tag{33}$$

By the assumption of unit elasticity of demand for the representative good in market i (recall we unit price elasticity of demand, $\eta = 1$, in eq.(21))

$$E(p_i) \equiv \overline{p_i} = p. \tag{34}$$

Hence on the right-side of eq.(33) it must be true that the sum of the terms after the first – after the aggregate price term p – have zero unconditional expectation

$$E\left[-\frac{(\gamma - \alpha)}{\gamma} \ln A + \frac{(\gamma - \alpha)}{\gamma} q + \frac{(\gamma - \alpha)}{\gamma} e_i^D - e_i^S\right] = 0. \tag{35}$$

Since e_i^D and e_i^S are random shocks to local market demand and supply, respectively, with zero expected value, we have

$$E\left[\frac{(\gamma - \alpha)}{\gamma} q\right] = \frac{(\gamma - \alpha)}{\gamma} \ln A \tag{36}$$

$$\Rightarrow E(q) = \ln A. \tag{37}$$

In this motivation of supply, $E(q)$ is clearly the aggregate potential or "natural" output level, $q^*_{(t)}$, of eq.(1), which corresponds to log output level produced when the macroeconomy on its balanced growth path.[10]

2.5 ***Notional and Behavioral Local Market Supply***

Consistent with q_i^S in (30) and q_t^* in (37), *notional* supply in the Lucas setup is

$$q_{it}^S = q_t^* + \beta \cdot (p_{it} - p_t) + \frac{\gamma}{(\gamma - \alpha)} e_{it}^S, \tag{38}$$

where $\beta = \frac{\alpha}{(\gamma-\alpha)} > 0$ is a composite supply parameter composed of the "deep", fundamental parameters of production and utility. Eq.(38) is the equation that producers would love to use to make production decisions; they'd like to jack up output when relative prices are running in their favor ("to make hay while the sun shines" as Americans say), and take leisure when relative price developments are running against them.

But, as noted in the 'state of the world', producers are not sure what the aggregate price level, p, is instantaneously. Hence they are not sure whether a movement in p_{it} represents a relative price shift or just inflation of the general price level. Below we shall see that *behavioral* supply function that actually drives production decisions is the ex-ante rational expectation of eq.(38):

$$q_{it}^S = q_t^* + \widetilde{\beta} \cdot [p_{it} - E(p_t|I_{it})] \tag{39}$$

where $\widetilde{\beta}$ is a parameter to be derived shortly, and $E(p_t|I_{it})$ is the expectation of p_t conditioned on the information available to producers in market i at decision time t.

What generates rationally $E(p_t|I_{it})$? Let's write down explicitly what producers know:

- $p_{it} = p_t + r_{it}, \qquad r_{it} \equiv (p_{it} - p_t)$; local market price is by definition general price plus a relative price shock, r_{it}.
- $p_t = E(p_t|I_{it}) + \varepsilon_t^P \sim 0,\ \sigma^2_{\epsilon^P},\ normal,\ white$; producers do not make systematic mistakes over time – producers expectations are rational.

[10] Hence, if as earlier,

$$q_t^* = \ln[A_0 \cdot F(K^*, N^*)] + \ln(1+g) \cdot t$$

then the ("deep") parameters of production and utility, α and γ, underly the convergent stocks of K and N and the rate of secular growthln $(1+g)$.

- Since $r_{it} \equiv (p_{it} - p_t) = \left\{p_{it} - \left[E(p_t|I_{it}) + \varepsilon_t^P\right]\right\} \sim 0,\ \sigma_r^2,\ normal,\ white,$ $\left(r_{it} + \varepsilon_t^P\right) = \left[p_{it} - E(p_t|I_{it})\right]$; producers know that expectation errors in gauging the relative price shock of interest will equal the sum of relative price shocks and aggregate price shocks.

- $Cov\left(r_{it}, \varepsilon_t^P\right) = 0$; by plausible assumption of the setup (by producers' historical observation), relative and general price shocks have zero covariance.

So how do the producers estimate r_{it} – the relative price movement they want to regulate supply with – from the observed deviation of p_{it} from expected p_t,which they know is equal to the composite error $\left(r_{it} + \varepsilon_t^P\right)$? This question is known as the 'signal extraction problem' – a commonplace concept among electrical engineers since the 1940s, but quite a novelty in economics when Lucas was writing back in the early 1970s.

One way to approach the signal extraction problem is to assume that producers have a quadratic loss function, that is, at each decision period they want to minimize $(r_i - \widehat{r_i})^2$. This would lead directly to a least-squares computation of the best linear unbiased (quadratic loss minimizing) predictor from historical data, that is from data in periods $t' < t$:[11]

$$\begin{aligned} r_{it'} &= b \cdot \left(r_{it'} + \varepsilon_{t'}^P\right) \\ &= b \cdot \left\{p_t - \left[p_{it} - E(p_t|I_{it})\right]\right\} \end{aligned} \tag{40}$$

$$\begin{aligned} b_{OLS} &= \frac{Cov\left[r_{it'}, \left(r_{it'} + \varepsilon_{t'}^P\right)\right]}{Var\left(r_{it'} + \varepsilon_{t'}^P\right)} \\ &= \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2} + \widehat{\sigma_{\varepsilon^P}^2}}, \qquad t' < t. \end{aligned} \tag{41}$$

Hence at decision period t producers would estimate the relative price movement of interest, r_{it}, from the observed composite expectation deviation $\left(r_{it} + \varepsilon_t^P\right) = \left[p_{it} - E(p_t|I_{it})\right]$ by computing

$$\begin{aligned} \widehat{r_{it}} &= \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2} + \widehat{\sigma_{\varepsilon^P}^2}} \cdot \left(r_{it} + \varepsilon_t^P\right) \\ &= \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2} + \widehat{\sigma_{\varepsilon^P}^2}} \cdot \left[p_{it} - E(p_t|I_{it})\right]. \end{aligned} \tag{42}$$

[11] Since the expected values of $r_{it'}$ and $\varepsilon_{t'}^P$ are both zero and they have zero covariance, one may in principle omit the constant (intercept) from the linear projection equation: constant $= \overline{r_{it'}} - b \cdot \overline{(r_{it'}) + \varepsilon_{t'}^P} = 0$.

It follows that the rational local market *behavioral* supply function would be

$$\begin{aligned} q_{it}^S &= q_t^* + \beta \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2} + \widehat{\sigma_{\varepsilon P}^2}} \cdot [p_{it} - E(p_t|I_{it})] \qquad (43) \\ &= q_t^* + \widetilde{\beta} \cdot [p_{it} - E(p_t|I_{it})], \end{aligned}$$

where $\beta \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2}+\widehat{\sigma_{\varepsilon P}^2}}$ is what I labelled $\widetilde{\beta}$ when I posited the behavioral supply function at eq.(39).

A second way to rationalize $\widetilde{\beta} = \beta \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2}+\widehat{\sigma_{\varepsilon P}^2}}$ is to forget about assuming a quadratic loss function and just apply a theorem from statistics (Cf. Mood, Graybill and Boes, many editions, chapter 5) which shows that if r_{it} and p_{it} are jointly normally distributed, the conditional expectation of r_{it} given realization of p_{it} is $\frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2}+\widehat{\sigma_{\varepsilon P}^2}} \cdot [p_{it} - E(p_t|I_{it})]$. Either way, one gets to the result in eq.(43). I like the former motivation, as it does not depend on joint distribution assumptions (which were assumed along the way in the setup, with appeal to this theorem in mind), though of course the rationale I favor does impute a loss function to the agents. Lucas used the joint normality projection, and most of the sorcerer's apprentices seem to have followed suit.

Taking the mean (unconditional expectation) of eq.(43) across local markets at time t gives the operative *Aggregate Supply* function[12]

$$q_t^S = q_t^* + \beta \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2} + \widehat{\sigma_{\varepsilon P}^2}} \cdot [p_t - E(p_t|I_t)]. \qquad (44)$$

Note well the implications of this aggregate supply schedule. [*Graph of LRAS, SRAS*] As general price variability gets large by comparison to relative price variability, producers no longer respond very much to 'surprising' price movements – price surprises that in a regime of low general price variability (low relative to historical calibrations of relative price variability) would elicit changes to production.

As $\frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2}+\widehat{\sigma_{\varepsilon P}^2}} \to 0$, that is, when all price movements historically originate with the general price level, the supply schedule is vertical in the price-output space, intersecting the output axis at the natural rate, q_t^*. By contrast as $\frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2}+\widehat{\sigma_{\varepsilon P}^2}} \to 1$,

[12] Notice that by substracting p_{t-1} from each price term within brackets in eq.(44) we obtain output supply (equal to actual output at equilibrium) in terms of the gap between actual and expected rates of inflation:
$q_t = q_t^* + \widetilde{\beta} \cdot [(p_t - p_{t-1}) - (E(p_t|I_t) - p_{t-1})]$. This is analogoust to the Phillips curve of eq.(16).

that is, in a historical regime of zero general price inflation in which all price movements are relative price shifts, producers will respond fully to an unexpected price movement in proportion $\beta = \frac{\alpha}{(\gamma-\alpha)}$ per unit price surprise.

The ratio of relative to general price variability is clearly contingent on the monetary policy-regime. This statement will become (more) obvious below.

2.6 *Aggregate Equilibrium*

The aggregate price level is endogenous; money (the generic aggregate demand policy variable of the model) is the only exogenous variable in this stylized macroeconomy. We find aggregate equilibrium in the usual way by equating supply to demand:

$$q_t^S = q_t^D \ \ \} \Rightarrow q_t \tag{45}$$

$$q_t^* + \widetilde{\beta} \cdot [p_{it} - E(p_t|I_t)] = (m_t - p_t) \ \ \} \Rightarrow q_t, \tag{46}$$

where recall $\widetilde{\beta} = \left[\beta \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2 + \sigma_{\varepsilon^P}^2}}\right] = \left[\frac{\alpha}{(\gamma-\alpha)} \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2 + \sigma_{\varepsilon^P}^2}}\right]$. You may readily confirm that the solutions for p_t and q_t are

$$p_t = \frac{1}{\left(1+\widetilde{\beta}\right)} m_t + \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} E(p_t|I_t) - \frac{1}{\left(1+\widetilde{\beta}\right)} q_t^* \tag{47}$$

$$q_t = \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} m_t - \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} E(p_t|I_t) + \frac{1}{\left(1+\widetilde{\beta}\right)} q_t^*. \tag{48}$$

Eq.(47) is the ex-post solution for realized aggregate price. Ex-ante, before p_t is realized, it must be true by rationality of expectations that the equation for ex-post equilibrium price holds in conditional expectation[13]:

$$E(p_t|I_t) = \frac{1}{\left(1+\widetilde{\beta}\right)} E(m_t|I_t) + \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} E(p_t|I_t) - \frac{1}{\left(1+\widetilde{\beta}\right)} q_t^*. \tag{49}$$

It follows that

$$\begin{aligned} E(p_t|I_t) \cdot \left[1 - \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} E(p_t|I_t)\right] &= \frac{1}{\left(1+\widetilde{\beta}\right)} E(m_t|I_t) - \frac{1}{\left(1+\widetilde{\beta}\right)} q_t^* \\ E(p_t|I_t) &= E(m_t|I_t) - q_t^*. \end{aligned} \tag{50}$$

[13] Remember that agents are assumed to know the natural output level, q_t^* along with parameters of utility and production.

Using the equations for realized aggregate price(eq. 47) and expected aggregate price (eq. 50), we see that the solution for p_t can be written

$$p_t = \frac{1}{\left(1+\widetilde{\beta}\right)} m_t + \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} \left[E(m_t|I_t) - q_t^*\right] - \frac{1}{\left(1+\widetilde{\beta}\right)} q_t^* \tag{51}$$

$$p_t = \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} E(m_t|I_t) + \frac{1}{\left(1+\widetilde{\beta}\right)} m_t - q_t^*. \tag{52}$$

The equilibrium solution for output in eq.(48) therefore is

$$q_t = \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} m_t - \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} \left[E(m_t|I_t) - q_t^*\right] + \frac{1}{\left(1+\widetilde{\beta}\right)} q_t^* \tag{53}$$

$$q_t = q_t^* + \frac{\widetilde{\beta}}{\left(1+\widetilde{\beta}\right)} \cdot \left[m_t - E(m_t|I_t)\right]. \tag{54}$$

Equation (54) just maps the earlier result for price surprises back onto money – the exogenous policy instrument in the stylized macroeconomy. The interpretation is therefore precisely the same: Only monetary surprises (or, more generally, surprising aggregate demand shifts) can generate deviations of log output from its "natural" level. And under the strict interpretation of the Lucas model, even the effect of monetary surprises will be damped monotonically as the ratio of relative price variability to general price variability falls. (Recall $\widetilde{\beta} = \left[\frac{\alpha}{(\gamma-\alpha)} \cdot \frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2+\sigma_{\varepsilon P}^2}}\right]$). Hence, there is no scope for systematic, and consequently predictable, monetary policy to affect the real economy. This is the core of the famous Lucas-Sagent-Wallace "*policy neutrality*" proposition, and you should understand its logical foundation thoroughly.

Lucas' "isolated markets" story motivating policy neutrality is, of course, preposterous. If information about the general price level were of importance to producers, or to any anyone else for that matter, it could be supplied at almost no cost at more or less the same frequency as asset price quotations. Yet the main message of the story has considerable merit. Agents in the economy (the agents of principals) surely do take pains to avoid being fooled by general price inflation and the monetary expansions that create it. The famous essay by Lucas developing the neutrality proposition in several policy and forecasting settings (the 'Lucas critique') is still perhaps the best place to learn about its range and power (Carnegie-Rochester series, 1976).

But all is not lost for policy activism. You should familiarize yourself with the 'older' new Keynesian models that give the authorities some capacity to affect the real economy because of institutionally determined sluggishness of wage adjustment, (leading examples are Fischer, JPE, 1977 and Taylor, 1979 AER, 1979), as well as subsequent new Keynesian models that look to inertia in product pricing (for example, Mankiw, QJE, 1985 and Ball et al. BPEA, 1988).

Finally, note that one could trace the $\frac{\widehat{\sigma_r^2}}{\widehat{\sigma_r^2}+\widehat{\sigma_{\varepsilon^P}^2}}$ component of the composite aggregate supply parameter $\widetilde{\beta}$ back to the volatility of money (volatility of aggregate demand policy) because, aside from potential output, it is money that ultimately drives the price level (eq.52).