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Consumption II: Ricardian Equivalence, Precautionary Saving and Habit Persistence

1 Government Spending and Intertemporal Consumption ("Ricardian Equivalence")

In earlier lectures consumption reacted to disposable income, that is, to private income less net taxes. Let's now put government fiscal action directly into the model. We will assume utility of consumption is based on "effective consumption," here defined as the weighted sum of private consumption, c, and the per consumption agent supply of government goods and services,[1] g:

$$c_t^* = c_t + \gamma g_t, \qquad \gamma \in (0,1)\,. \tag{1}$$

Hence agents now act to maximize a utility program with time separable preferences over private consumption and the weighted flow of goods and services from government

$$\underset{\left\{0<c^*_{t+j}\right\}_{j=0}^{\infty}}{Max} \; E_t \sum_{j=0}^{\infty} \beta^j u\left({c^*}_{t+j}\right) \tag{2}$$

subject to the usual solvency and budget constraints

$$\lim_{j\to\infty} \left(\frac{1}{R}\right)^j a_{t+j} = 0 \tag{3}$$

$$\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t+j} = \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j \left(w_{t+j} - tx_{t+j}\right) + a_t \tag{4}$$

[1] Note that we are considering government supplied goods and services, not cash transfers to households.

where note that I keep the interest rate factor, $R = (1 + r)$, constant, and disposable income – denoted earlier by qd – now explicitly distinguishes labor and private transfer income, w, and net taxes, tx; $qd \equiv (w - tx)$. Note also that the linear effective consumption equation (1) means that the marginal rate of substitution between c and g is constant, and it implies that a unit of per agent government goods and services has the same value to effective consumption as γ units of private consumption.

With this setup we aim to learn (i) how sensitive private consumption (demand) is to tax versus debt finance of government current expenditure, and (ii) whether an increase in government spending produces an offsetting decline in private consumption expenditure ex-ante.

Let B be the government's debt of one-period maturity. Given the government's solvency constraint

$$\lim_{j \to \infty} \left(\frac{1}{R} \right)^j B_{t+j} = 0, \tag{5}$$

the difference equation for debt accumulation

$$B_{t+1} = R \cdot (B_t + G_t - TX_t), \tag{6}$$

implies that the the government's intertemporal budget constraint is[2]

$$\sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j TX_{t+j} = \sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j G_{t+j} + B_t, \tag{7a}$$

which I shall write on a per consumption agent basis as

$$\sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j tx_{t+j} = \sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j g_{t+j} + b_t \tag{8}$$

where TX and tx denote gross and per agent tax revenues, respectively, and B and b denote gross and per agent government debt, respectively.[3]

[2] The quickest way to find eqs.(6) – ($7a$) is by application of lag/lead algebra, just as I did earlier in these lectures to find the consumer's intertemporal budget constraint.

[3] To keep things simple I hold the number of agents constant (no population growth). Have a look again at the Ramsey-Cass-Koopmans lecture to see the small additional complication

Agents are forward looking and capitalize the future tax obligations implied by government's issuance of debt, while at the same time taking consumption benefit from the value of government goods and services (remember we have $u(c^*)$ for period utilities). Hence forward looking 'effective' consumers have an 'effective' budget constraint that is written in terms of c^* and b. Adding the PDV of $\gamma \cdot g$ to the left- and right-sides of the consumers budget constraint in (4), and substituting for the PDV of tx from (8), gives the consumer's effective budget constraint as

$$\begin{aligned}
\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j}(c_{t+j}+\gamma g_{t+j}) &= \sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j}(w_{t+j}+\gamma g_{t+j}) \qquad (9)\\
&\quad -\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j} tx_{t+j}+a_t \\
\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j}(c_{t+j}+\gamma g_{t+j}) &= \sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j}(w_{t+j}+\gamma g_{t+j}-g_{t+j}) \\
&\quad -b_t+a_t \qquad (10)
\end{aligned}$$

$$\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j} c^*_{t+j}=\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^{j}\left[w_{t+j}+(\gamma-1)\cdot g_{t+j}\right]+(a_t-b_t)\,. \qquad (11)$$

It is now clear that the PDV of effective consumption, $c^* = c_t + \gamma g$, is constrained by net society-wide wealth per agent, $(a_t - b_t)$ plus the PDV of gross incomes, w, plus $(\gamma - 1)$ times the PDV of government expenditure, g. Note the implication of the later term: If the parameter that weights government consumption benefits relative to private consumption benefits satisfies $\gamma < 1$ (that is if a unit of government supplied goods and services is valued less than a unit of private consumption), then the PDV of government spending amounts to a negative wealth effect on optimal consumption.

that arises with a growing number of consumers, which amounts to carrying another another parameter, the population growth rate "n," *if we* abstract from fertility choice issues and impose a constant rate of population growth. Note also that taxes and government benefits are implicitly distributed uniformly over the population of consumers – an unrealistic assumption imposed to make the main points without distributional complications. For a penetrating critique of such representative agent simplifications, see Kirman, J.Econ.Perspectives, 1997.

Using techniques you already have well at hand, we know that optimal intertemporal 'effective' consumption must satisfy the Euler equation

$$u'\left(c_t^*\right) = E_t \frac{(1+r)}{(1+\rho)} u'\left(c_{t+1}^*\right). \tag{12}$$

1.1 Closed Form Results

To get a closed form solution, and thereby see what can be learned about fiscal policy effects on the consumption function proper, we must commit to a utility function. Let's keep things simple, and at the same time permit direct comparison to Hall's random walk result. So assume $\rho = r$ and quadratic utility in c^*

$$u(c_t^*) = -\frac{1}{2}\left(\bar{c}^* - c_t^*\right)^2, \tag{13}$$

which implies via the Euler equation

$$E_t c_{t+j}^* = c_t^*, \qquad j > 0. \tag{14}$$

Now we can find the consumption function from the budget constraint in eq.(11) by taking time t conditional expectations:

$$E_t \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t+j}^* = E_t \left\{ \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j \left[w_{t+j} + (\gamma - 1) \cdot g_{t+j}\right] + (a_t - b_t) \right\}. \tag{15}$$

Since $E_t c_{t+j}^* = c_t^*$, the left-side of (15), as previously, is

$$E_t \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t+j}^* = E_t \left(\frac{c_t^*}{1 - \frac{1}{R} L^{-1}} \right) = \frac{(1+r)}{r} c_t^*, \tag{16}$$

and since $a_t,\ b_t$ are known at time t, the 'effective consumption function' is

$$\frac{(1+r)}{r} c_t^* = E_t \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j \left[w_{t+j} + (\gamma - 1) \cdot g_{t+j}\right] + (a_t - b_t) \tag{17}$$

$$
\begin{aligned}
c_t^* &= \frac{r}{(1+r)} \cdot \left\{ E_t \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j [w_{t+j} + (\gamma - 1) \cdot g_{t+j}] + (a_t - b_t) \right\} \qquad (18) \\
&= \frac{r}{(1+r)} \cdot \{ E_t PDV \;\; [w_{t+j} + (\gamma - 1) \cdot g_{t+j}] + (a_t - b_t) \}
\end{aligned}
$$

Given that effective consumption is $c_t^* = c_t + \gamma g_t$, we need to move the private consumption value of government spending γg_t to the right-side in order to get observable consumption on the left:

$$
c_t = \frac{r}{(1+r)} \cdot \{ E_t PDV \, [w_{t+j} + (\gamma - 1) \cdot g_{t+j}] + (a_t - b_t) \} - \gamma g_t. \qquad (19)
$$

The consumption function of eq.(19) clearly implies that current private consumption spending declines at the annuity cost of the current addition to the stock of public debt, that is

$$
\frac{\partial c_t}{\partial b_t} = - \frac{r}{(1+r)},
$$

which means that debt finance does not add to consumer wealth. This is the core of the Ricardian equivalence proposition (introduced to contemporary economics in Robert Barro's celebrated 1974 JPE paper "Are Government Bonds Net Wealth"): Government debt induces full ex-ante crowding out of private consumption expenditure. Consumers understand that as taxpayers they eventually must foot the expenditure bills, and they regulate private consumption and saving accordingly – a conclusion that follows here as an accounting matter from the behavioral assumption that private agents fully capitalize the government's budget constraint to their utility maximization programs.

The decline in private consumption spending induced by changes to government's supply of goods and services, however, is less than one-for-one as long as

$\gamma < 1$.

$$\begin{aligned}\frac{\partial c_t}{\partial g_t} &= \frac{\partial\left[\frac{r}{(1+r)}\cdot(\gamma-1)-\gamma\right]g_t}{\partial g_t}\\ &= -\frac{(\gamma+r)}{(1+r)}.\end{aligned}$$

If $\gamma < 1$, a Euro of extra g does not induce a correspondingly large reduction to private consumption expenditure. This gives some scope for fiscal policy to expand aggregate demand by increasing g during, say, periods of output contraction.

The interesting question is whether or not the Ricardian equivalence proposition holds empirically. You should be familiar with some of the empirical literature; yet you might think afresh about how the matter might be productively investigated. Among other things, empirics would require a model for the evolution of labor incomes and the evolution of government spending.

As for government spending on goods and services alone, we could learn something about its weight in utility of effective consumption from the Euler equation. With quadratic utility ("certainty equivalence"), we know that the Euler implies

$$E_t c^*_{t+1} = c^*_t$$

so that

$$E_t c_{t+1} = c_t + \gamma g_t - \gamma E_t g_{t+1} \tag{20}$$

$$c_{t+1} = c_t - \gamma\cdot(g_{t+1} - g_t) + e_{t+1}, \tag{21}$$

where

$$e_{t+1} = [(c_{t+1} - E_t c_{t+1}) + \gamma\cdot(g_{t+1} - E_t g_{t+1})] \sim 0,\ \textit{white}.$$

Estimation of a stochastic consumption equation based on equations like (21)– you of course need not impose $\rho = r$ – could yield some evidence about the utility value of government supplied goods and services relative to private consumption choices.

2 Precautionary Saving

How do changes in uncertainty about future incomes – in particular a change in the expected variance of incomes – affect consumption behavior? Given a linear budget constraint, the answer to this question turns out to depend on the curvature of the utility function. As you already know, if marginal utility is linear in consumption, uncertainty about incomes does not affect expected marginal utility and, therefore, has no effect on optimal consumption behavior. Hence the "certainty equivalence" property of quadratic utility: Optimal behavior is effected only by expected income, not its higher moments (e.g. variance).

Recall that risk averse agents prefer a more certain lower-payoff outcome to a less certain potentially higher payoff one. One standard measure is the Arrow-Pratt coefficient of relative risk aversion (CRRA):

$$CRRA = -\frac{u''(c)}{u'(c)} \cdot c \left\} \begin{array}{l} > 0 \text{ risk aversion} \\ = 0 \text{ risk neutrality} \\ < 0 \text{ risk preferring} \end{array} \right. \tag{22}$$

another is the coefficient of absolute risk aversion (CARA):

$$-\frac{u''(c)}{u'(c)} \tag{23}$$

When marginal utility is not linear, that is, when $u'''(c) \neq 0$, uncertainty affects consumption behavior – perhaps profoundly. Plausible utility functions have $u'''(c) > 0$, which means that marginal utility is convex in consumption, and increases to uncertainty raise expected marginal utility. This implies that consumption is moved forward (to the future) in time, raising the incentive to save today ("prudence").

2.1 An Illustration Using the Euler Equation for CIES-CRRA Utility

First consider an illustration of precautionary saving based on analysis of the Euler equation for optimal intertemporal consumption when utility is of CIES-CRRA form. Period utilities have the properties[4]

$$
\begin{aligned}
u(c) &= \frac{c^{1-\frac{1}{s}}-1}{1-\frac{1}{s}}, \qquad 0 \neq s > 0 && (24)\\
u'(c) &= c^{-\frac{1}{s}} > 0 && (25)\\
u''(c) &= -\frac{1}{s}c^{-\left(1+\frac{1}{s}\right)} < 0 \qquad \text{utility is concave} && (26)\\
u'''(c) &= \frac{1+s}{s^2}c^{-\left(2+\frac{1}{s}\right)} > 0 \qquad \text{marginal utility is convex} && (27)
\end{aligned}
$$

and the coefficients of relative and absolute risk aversion imply risk averse agents

$$
\begin{aligned}
CRRA &= -\frac{u''(c)}{u'(c)} \cdot c && (28)\\
&= -\frac{-\frac{1}{s}c^{-\left(1+\frac{1}{s}\right)}}{c^{-\frac{1}{s}}}\\
&= \frac{1}{s} > 0
\end{aligned}
$$

$$
\begin{aligned}
CARA &= -\frac{u''(c)}{u'(c)} && (29)\\
&= \frac{1}{c \cdot s} > 0 && (30)
\end{aligned}
$$

– marginal utility is convex with positive third derivative (eq.27). [Graph]

Recall the Euler equation for utility maximization is

$$
\begin{aligned}
u'(c_t) &= E_t\left\{\beta \cdot R_{t+1} \cdot u'(c_{t+1})\right\}\\
\beta &\equiv \frac{1}{(1+\rho)}, \; R_{t+1} \equiv (1 + r_{t+1})
\end{aligned}
$$

which in the CIES/CRRA case (eq.24) is

$$
c_t^{-\frac{1}{s}} = E_t\left\{\beta \cdot R_{t+1} \cdot c_{t+1}^{-\frac{1}{s}}\right\}. \tag{31}
$$

[4] The important special case of log utility is given by $\lim_{s\to 1} \frac{c^{1-\frac{1}{s}}-1}{1-\frac{1}{s}} = \ln c$.

Now divide through by $c_t^{-\frac{1}{s}}$ and use an exponential-log representation to obtain

$$\begin{aligned}
1 &= E_t \exp\left[\ln\left(\beta \cdot R_{t+1} \cdot c_{t+1}^{-\frac{1}{s}} \cdot c_t^{\frac{1}{s}}\right)\right] \\
1 &= E_t \exp\left[-\rho + r_{t+1} - \frac{1}{s}\ln\left(c_{t+1}/c_t\right)\right] \qquad (32) \\
&= E_t \exp\left[-\rho + r_{t+1} - \frac{1}{s}\Delta \ln c_{t+1}\right]
\end{aligned}$$

where $-\rho = \ln\beta = \ln\frac{1}{1+\rho}$, $r_{t+1} = \ln R_{t+1} = \ln\left(1 + r_{t+1}\right)$.

Next assume that $\Delta \ln c_{t+1}$ is conditionally normally distributed with variance σ_{t+1}^2, which implies that[5]

$$E_t\left[\exp\left(-\frac{1}{s}\Delta \ln c_{t+1}\right)\right] = \exp\left[-\frac{1}{s}E_t\Delta \ln c_{t+1} + \frac{1}{2s^2}\sigma_{t+1}^2\right]. \qquad (33)$$

Substituting (33) into (32) gives

$$1 = \exp\left[-\rho + E_t r_{t+1} - \frac{1}{s}E_t\Delta \ln c_{t+1} + \frac{1}{2s^2}\sigma_{t+1}^2\right]. \qquad (34)$$

Taking logs and solving for $E_t\Delta \ln c_{t+1}$ yields

$$\begin{aligned}
0 &= -\rho + E_t r_{t+1} - \frac{1}{s}E_t\Delta \ln c_{t+1} + \frac{1}{2s^2}\sigma_{t+1}^2 \\
\frac{1}{s}E_t\Delta \ln c_{t+1} &= -\rho + E_t r_{t+1} - \frac{1}{s}E_t\Delta \ln c_{t+1} + \frac{1}{2s^2}\sigma_{t+1}^2 \\
&\Rightarrow E_t\Delta \ln c_{t+1} = s\left(E_t r_{t+1} - \rho\right) + \frac{1}{2s}\sigma_{t+1}^2. \qquad (35)
\end{aligned}$$

The implied relation for observed changes in log consumption is therefore

$$\Delta \ln c_{t+1} = s\left(E_t r_{t+1} - \rho\right) + \frac{1}{2s}\sigma_{t+1}^2 + e_{t+1}, \quad e_{t+1} \sim 0,\ \textit{white}. \qquad (36)$$

The conditional - time varying variance term in (36) is known as the "precautionary savings" term. It implies that as the variance of shocks to consumption rises (owing to increased variance of shocks to underlying incomes), the growth

[5] The result in (33) may be found in many statistics texts.

rate of consumption will ceteris paribus tend to be more positively sloped in time. In other words, increases to uncertainty induce less consumption and more saving today as a hedge against the increased potential of future rainy days. If we evaluate an observation regime running, say, from an initial period 0 to period $t+1$ in which the interest rate was constant at $\bar{r}$ and the variance of shocks was constant at σ^2, the implied time path of log consumption would be

$$\ln C_{t+1} = \ln C_0 + s \cdot (\bar{r} - \rho) \cdot (t+1) + \frac{1}{2s}\sigma^2 \cdot (t+1) \, . \tag{37}$$

Note that the larger is the elasticity of substitution, s, the smaller is precautionary saving (the smaller is "relative prudence")[6] at any given variance of shocks, σ^2.

2.2 A Closed Form Solution for the CARA Utility Case

A simple case that permits a closed-form solution, which perhaps conveys more intuition about what is going on than a purely analytical demonstration (but at the cost of realism), is a setup based on the Constant Absolute Risk Aversion (CARA) utility function based on Caballero (JME, 1990).

2.2.1 Utility

$$u(c) = -\frac{1}{\alpha}\exp(-\alpha \; c_t) \, , \qquad \alpha > 0 \tag{38}$$

$$\begin{aligned} u'(c) &= \exp(-\alpha \; c_t) > 0 && (39) \\ u''(c) &= -\alpha \; \exp(-\alpha \; c_t) < 0 \qquad \text{utility is concave} && (40) \\ u'''(c) &= \alpha^2 \exp(-\alpha \; c_t) > 0 \qquad \text{marginal utility is convex} \end{aligned}$$

[6] The Arrow-Pratt measure of relative prudence for CIES/CRRA utility is

$$-\frac{u'''(c) \cdot c}{u''(c)} = \frac{1+s}{s}$$

which falls as s rises:

$$\frac{d\left(\frac{1+s}{s}\right)}{ds} = -\frac{1}{s^2}.$$

.

The curvature of utility implies constant "relative prudence" equal to α.[7]

2.2.2 The Consumer's Program

$$\underset{\{c_{t+j}\}_{j=0}^{T-t}}{Max} \; E_t \sum_{j=0}^{T-t} -\frac{1}{\alpha} \exp\left(-\alpha \; c_{t+j}\right) \tag{41}$$

subject to the simplified budget constraint $(r = \rho = 0)$

$$a_{t+1} = \left(a_t + qd_t - c_t\right), \tag{42}$$

a_t given and the assumption that incomes evolve as a random walk with normally distributed shocks (innovations to income):

$$qd_t = qd_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim Normal\left(0, \sigma^2\right). \tag{43}$$

The crucial assumption is that utility takes the CARA form;[8] the remaining assumptions are made for simplicity. Given the assumed process generating incomes, it follows that

$$c_{t+j}\}_{j=0}^{T-t} \sim Normal, \; \sigma^2. \tag{44}$$

[7] The Arrow-Pratt measure of relative prudence is

$$-\frac{u'''\left(c\right) \cdot c}{u''\left(c\right)} = \alpha.$$

[8] CARA utility is attractive because it enables a closed form solution. But there are problems: Optimal consumption in principle can be negative; high and low income households have the same MPC but different APCs; the intertemporal elasticity of substitution does not exist; everyone of the same age has the same stock of wealth regardless of income histories because consumption adjusts immediately and fully to (the permanent) innovations to income. See Irvine and Wang, AER,Dec. 1994.

The time t conditional expectation of marginal utility at $t+1$ therefore is[9]

$$\begin{aligned} E_t\left[\exp\left(-\alpha\ c_{t+1}\right)\right] &= \exp\left[E_t\left(-\alpha\ c_{t+1}\right)+\frac{1}{2}Var\left(-\alpha\ c_{t+1}\right)\right] \\ &= \exp\left[E_t\left(-\alpha\ c_{t+1}\right)+\frac{1}{2}\alpha^2\sigma^2\right]. \end{aligned} \tag{45}$$

The Euler equation for optimal consumption

$$E_t\left[u'\left(c_{t+1}\right)\right]=u'\left(c_t\right) \tag{46}$$

consequently takes the form

$$\exp\left[E_t\left(-\alpha\ c_{t+1}\right)+\frac{1}{2}\alpha^2\sigma^2\right]=\exp\left(-\alpha\ c_t\right). \tag{47}$$

Taking logs of the Euler we obtain

$$-\alpha\ E_t\left(c_{t+1}\right)+\frac{1}{2}\alpha^2\sigma^2 = -\alpha\ c_t \tag{48}$$

$$E_t\left(\ c_{t+1}\right) = c_t+\frac{1}{2}\alpha\sigma^2 \tag{49}$$

$$c_{t+1} = c_t+\frac{1}{2}\alpha\sigma^2+e_{t+1}, \qquad e_{t+1}\sim 0,\ white. \tag{50}$$

Iterating one finds

$$c_{t+j} = c_t+j\cdot\left(\frac{1}{2}\alpha\sigma^2\right)+\sum_j e_{t+j} \tag{51}$$

$$E_t\left(\ c_{t+j}\right) = c_t+j\cdot\left(\frac{1}{2}\alpha\sigma^2\right), \qquad j=0,1,...,T-t. \tag{52}$$

[9] As noted before, if x is normally distributed with mean $E[x]$ and variance σ^2, then

$$E\left[\exp\left(x\right)\right]=\exp\left(E\left[x\right]+\frac{1}{2}\sigma^2\right).$$

Hence the larger the variance of innovations to income (the greater is uncertainty), σ^2, and the greater the coefficient of relative prudence in the utility of consumption function, α, the more upward sloping will be the optimal consumption plan, and the greater will be precautionary saving early in life.

2.2.3 Uncertainty and the Consumption Function

The income process

$$qd_{t+j} = qd_{t+j-1} + \varepsilon_{t+j}, \qquad j \succeq 0$$

implies that

$$qd_{t+j} = qd_t + \sum_{j=1}^{T-t} \varepsilon_{t+j}. \tag{53}$$

Note that innovations to income *permanently* affect income in a random walk process. Hence qd_{t+j} is permanent income at $t + j$.

The budget constraint

$$a_{t+1} = (a_t + qd_t - c_t) \tag{54}$$

implies

$$\begin{aligned} c_t &= a_t \left(1 - L^{-1}\right) + qd_t \qquad (55) \\ \Rightarrow & \frac{c_t}{(1 - L^{-1})} = a_t + \frac{qd_t}{(1 - L^{-1})}. \end{aligned}$$

Taking time t expectations and evaluating forward T periods gives

$$E_t \sum_{j=0}^{T-t} c_{t+j} = E_t \sum_{j=0}^{T-t} qd_{t+j} + a_t. \tag{56}$$

Given (52) and (53) the budget constraint (56) can be expressed

$$
\begin{aligned}
(T-t+1)\cdot c_t + \sum_{j=0}^{T-t} j\cdot\left(\frac{1}{2}\alpha\sigma^2\right) &= a_t + (T-t+1)\cdot qd_t + E_t\sum_{j=1}^{T-t}\varepsilon_{t+j} \\
(T-t+1)\cdot c_t + \sum_{j=0}^{T-t} j\cdot\left(\frac{1}{2}\alpha\sigma^2\right) &= a_t + (T-t+1)\cdot qd_t. \qquad (57)
\end{aligned}
$$

Consumption at any time t can therefore be written[10]

$$
\begin{aligned}
c_t &= \frac{1}{(T-t+1)}\left[a_t - \sum_{j=0}^{T-t} j\cdot\left(\frac{1}{2}\alpha\ \sigma^2\right)\right] + qd_t \\
c_t &= \frac{1}{(T-t+1)}a_t + qd_t - \frac{1}{4}(T-t)\,\alpha\ \sigma^2. \qquad (58)
\end{aligned}
$$

We have arrived finally at the punch line: Income uncertainty σ^2 reduces current consumption (increases current saving) at a rate that diminishes as $t \to T$, that is, as the end of life approaches. At $t = T$

$$
c_T = a_T + qd_T \qquad (59)
$$

and a_T will be higher than in the quadratic utility case because of higher saving earlier in life.[11]

[10] In eq.(58) I make use of the fact that

$$
\begin{aligned}
\frac{1}{(T-t+1)}\sum_{j=0}^{T-t} j\cdot\left(\frac{1}{2}\alpha\sigma^2\right) &= \frac{1}{2}(T-t)\left(\frac{1}{2}\alpha\sigma^2\right) \\
&= \frac{1}{4}(T-t)\,\alpha\ \sigma^2
\end{aligned}
$$

which you can easily verfiy.

[11] Note that if the utility function were quadratic

$$
u(c) = -\frac{1}{2}(\bar{c}-c)^2
$$

and everything else in the setup was the same, the consumption function would be

$$
c_t = \frac{1}{(T-t+1)}a_t + qd_t
$$

3 Reference Dependent Utility of Consumption: Habit Persistence

The traditional utility of consumption functions with time separable period utilities $u(c_t)$ has not fared well in empirical analyses. This led to relaxation of time separability and exploration of reference dependent utility setups. Here I consider a popular, simple alternative known as the "habit persistence" model.[12]

Assume a standard utility maximization program

$$\underset{\{c^*_{t+j}\}_{j=0}^{\infty}}{Max} \; E_t \sum_{j=0}^{\infty} \beta^j u\left(c^*_{t+j}\right) \tag{60}$$

subject to the usual infinite horizon intertemporal budget constraint (with fixed interest rate)

$$\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t+j} = \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j q d_{t+j} + a_t \tag{61}$$

where c^* denotes a reference dependent consumption variate. A simple form of reference dependence – the first-order habit persistence model – which can be traced in a general way back to (the increasingly appreciated) work of James Duesenberry, Income, Saving and the Theory of Consumer Behavior, 1949, and even to Thorstein Veblen's, Conspicuous Consumption, 1899 – is

$$c^*_t = c_t - \gamma c_{t-1}, \qquad \gamma \in (0,1). \tag{62}$$

The idea is that contemporaneous utility of consumption depends upon deviations of current consumption from consumption the previous period – specifically it's a lot easier to experience Instead of additive separability in c, we now have additive separability in c^* in a fashion analogous to the Ricardian equivalence setup presented earlier in these notes. More complicated and more interesting reference standards, including outside consumption standards external to the agents own

which is identical to the CARA case at $t = T$ (uncertainty expires with life!).

[12] The 2015 Nobel Memorial Prize in economics to Angus Deaton was partly motivated by his work on theoretical and empirical inconsistencies in the textbook (economics!) models of consumption.

consumption history, as in "keeping up with" or "ahead of" the Jones', might also be usefully entertained.

Re-writing the budget constraint in terms of c^* in (62) by subtracting $\gamma \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t-1+j}$ from the left- and right-sides of (61), we obtain

$$\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c^*_{t+j} = \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} + a_t - \gamma \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c_{t-1+j} \tag{63}$$

$$= \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} + a_t - \gamma \left(\begin{array}{c} c_{t-1} + R \cdot \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} \\ +a_t \end{array} \right) \tag{64}$$

$$= -\gamma c_{t-1} + \frac{(1+r-\gamma)}{(1+r)} \cdot \left(\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} + a_t \right). \tag{65}$$

In the quadratic utility case, with $r = \rho$, the Euler equation for optimal consumption is

$$E_t c^*_{t+j} = c^*_t, \tag{66}$$

which given the reference augmented budget constraint in (65) implies that the consumption function is

$$E_t \left[\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j c^*_{t+j} \right] = E_t \left[-\gamma c_{t-1} + \frac{(1+r-\gamma)}{(1+r)} \cdot \left(\begin{array}{c} \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} \\ +a_t \end{array} \right) \right] \tag{67}$$

$$\frac{(1+r)}{r} c_t - \gamma \frac{(1+r)}{r} c_{t-1} = -\gamma c_{t-1} + \frac{(1+r-\gamma)}{(1+r)} \cdot \left(\begin{array}{c} E_t \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} \\ +a_t \end{array} \right) \tag{68}$$

$$c_t = \frac{\gamma}{(1+r)} c_{t-1} + \left(1 - \frac{\gamma}{(1+r)}\right) \cdot qd^p_t \tag{69}$$

where

$$qd^p_t = \frac{r}{(1+r)} \cdot \left(E_t \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j qd_{t+j} + a_t \right). \tag{70}$$

Hence consumption is a weighted average of lagged consumption and contemporaneous permanent income. The larger is γ, the more persistent is optimal consumption. At $\gamma = 0$ consumption of course collapses to the standard PIH model

$$c_t = qd_t^p. \tag{71}$$

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