Douglas Hibbs
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Dynamic General Macroeconomic Equilibrium

This lecture runs through a bare bones model of dynamic general macroeconomic equilibrium (deterministic). During subsequent lectures many of the constituents of the model will receive sustained attention.

1 The state of the world

The state of the world is standard neoclassical in a Ramsey-Cass-Koopmans setting. The economy is composed of numerous, identical price taking households and firms. At every point in time three goods are traded: labor services N, capital K, and a single final output good, Q, which is exhaustively allocated to investment, I, or consumption, C. There is no government sector – at this point we abstract from monetary policy, fiscal policy and government purchases of goods and services (G). Time is discrete and the economy is deterministic (no stochastic shocks).

Output production (value added) is Cobb-Douglas:

$$Q_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{1}$$

where A_t is the exogenously given state of technology at each period, and $0 < \alpha < 1$.

The accumulation equation for the stock of capital (for simplicity we assume at this point that there is no depreciation) is:

$$\begin{aligned} K_{t+1} &= (Q_t - C_t) + K_t \qquad (2) \\ &= I_t + K_t. \end{aligned}$$

Households are infinitely lived ("dynasties"), and they value expected consumption and leisure streams according to the utility function:

$$U = E_{t=0} \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{(N_t^s)^{1+\phi}}{1+\phi} \right] \tag{3}$$

where $E_{t=0}$ denotes (rational) expectations conditioned on information available to all agents at time $t = 0$, $\beta = \frac{1}{(1+\rho)}$ is the discount factor, $\rho > 0$ is the rate of time preference, C is consumption, N^s is labor supplied by households to firms, and $\phi > 0$ is the elasticity of the *marginal utility* of time, that is

$$\begin{aligned} \phi &= \frac{\partial U'(N_t^s)/U'(N_t^s)}{\partial N_t^s/N_t^s} = \frac{\partial U'(N_t^s)}{\partial N_t^s} \cdot \frac{N_t^s}{U'(N_t^s)} \\ &= \frac{\partial \left(-\beta^t (N_t^s)^{\phi}\right)}{\partial N_t^s} \cdot \frac{N_t^s}{-\beta^t (N_t^s)^{\phi}} \\ &= \left(-\beta^t \phi (N_t^s)^{\phi-1}\right) \cdot \frac{N_t^s}{-\beta^t (N_t^s)^{\phi}} \\ &= \frac{\left(-\beta^t \phi (N_t^s)^{\phi}\right)}{-\beta^t (N_t^s)^{\phi}} = \phi. \end{aligned} \tag{4}$$

The corresponding elasticity of marginal utility of consumption is minus unity (a notable feature of log utility):

$$\begin{aligned} -1 &= \frac{\partial U'(C_t)/U'(C_t)}{\partial C_t/C_t} = \frac{\partial U'(C_t)}{\partial C_t} \cdot \frac{C_t}{U'(C_t)} \\ &= \frac{\partial \left(\beta^t C_t^{-1}\right)}{\partial C_t} \cdot \frac{C_t}{\beta^t C_t^{-1}} \\ &= -\beta^t C_t^{-2} \cdot \frac{C_t}{\beta^t C_t^{-1}} = -1 \end{aligned} \tag{5}$$

As we shall see later on, log utility of consumption implies that households are willing to substitute intertemporally point-for-point with deviations of the rate of time preference from the rate at which assets accumulate (the interest rate).

2 The Producer's Program

Firms choose demand for capital and labor at every point in time to maximize a one-period real profit function, π_t, taking the real wage, w, and the cost of capital, r, as given:

$$\begin{aligned}\pi_t &= Q_t - w_t N_t^d - r_t K_t^d \\ &= A_t K_t^\alpha N_t^{1-\alpha} - w_t N_t^d - r_t K_t^d. \end{aligned} \tag{6}$$

where A is the exogenously given state of technology (total factor productivity).

The FOC for labor demand is

$$\frac{\partial \pi_t}{\partial N_t^d} = 0 = (1-\alpha) A_t \left(K_t^d\right)^\alpha \left(N_t^d\right)^{-\alpha} - w_t \tag{7}$$

so that labor is deployed in production such that the given wage equals the marginal product of labor:

$$w_t = (1-\alpha) A_t \left(K_t^d\right)^\alpha \left(N_t^d\right)^{-\alpha}. \tag{8}$$

The FOC for capital demand is

$$\frac{\partial \pi_t}{\partial K_t^d} = 0 = \alpha A_t \left(K_t^d\right)^{\alpha-1} \left(N_t^d\right)^{1-\alpha} - r_t \tag{9}$$

so that capital is deployed in production such that the given cost of capital is equated to the marginal product of capital:

$$r_t = \alpha A_t \left(K_t^d\right)^{\alpha-1} \left(N_t^d\right)^{1-\alpha}. \tag{10}$$

3 The Consumer's Program

Households own the capital stock and are initially endowed with K_0 units along with $\overline{N}$ units of time per period. By assumption $\overline{N}$ is large enough not to be binding. (As in the case of firms, aggregation issues are fudged over and I proceed as if the representative household subsumes the entire household sector. I return to this matter in subsequent lectures.) Households choose C, N^s, K^s and I to maximize the intertemporal expected utility function

$$U\left(t=0\right) \Rightarrow Max_{\left\{C_t, N_t^s, K_{t+1}^s, I_t\right\}_{t=0}^{\infty}} E_{t=0} \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{\left(N_t^s\right)^{1+\phi}}{1+\phi}\right] \qquad (11)$$

subject to the time constraint

$$N_t^s \leq \overline{N}$$

and the budget constraint

$$\begin{aligned} I_t + C_t &= \left(K_{t+1}^s - K_t^s\right) + C_t \\ &= r_t K_t^s + w_t N_t^s = Q_t. \end{aligned}$$

Note that the second line of the budget constraint above follows from the assumed constant returns to scale (Cobb-Douglas) production; factor payments exhaust total value added:

$$\frac{\partial Q}{\partial K} K + \frac{\partial Q}{\partial L} L \equiv Q_K K + Q_L L = Q.$$

I will proceed here (but not in subsequent lectures) as if agents had perfect foresight, that is, $E_{t=0} X_t = X_t$ for any variable X at any time $t \succeq 0$. The conditional expectations operator $E_{t=0}$ is therefore without effect (arm-waving).

4 Optimal Consumption and Labor Supply

Let λ_t denote the Lagrangian multiplier on the time t budget constraint. The household's sequential decision problem can be posed as:

$$U(t=0) \Rightarrow Max_{\left\{C_t, N_t^s, K_{t+1}^s\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \ln C_t - \frac{(N_t^s)^{1+\phi}}{1+\phi} + \\ \lambda_t \cdot \left[r_t K_t^s + w_t N_t^s - \left(K_{t+1}^s - K_t^s\right) - C_t \right] \end{array} \right] \tag{12}$$

subject to the transversality condition

$$t \overset{\lim}{\rightarrow} \infty \beta^t \lambda_t K_t = 0$$

and the time constraint

$$N_t^s \leq \overline{N}.$$

Recall that there is no depreciation of capital and therefore in the household budget constraint $I_t = \left(K_{t+1}^s - K_t^s\right)$. The FOC for time t choice of consumption is

$$FOC\left(C_t\right) \Rightarrow \frac{\partial U}{\partial C_t} = \beta^t \left(\frac{1}{C_t} - \lambda_t\right) = 0 \tag{13}$$

implying

$$\lambda_t = \frac{1}{C_t}. \tag{14}$$

The value of the Lagrangian multiplier therefore equals the marginal utility of consumption at each period, since $U'\left(C_t\right) = \frac{1}{C_t}$.

The FOC for choice of next period's desired stock of capital (K_t^s is given at each decision period), K_{t+1}^s, is obtained by evaluating (12) at at periods t and $t+1$:

$$FOC\left(K_{t+1}\right) \Rightarrow \frac{\partial U\left(t=0\right)}{\partial K_{t+1}} + \frac{\partial U\left(t=1\right)}{\partial K_{t+1}} = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} \left(r_{t+1} + 1\right) = 0. \tag{15}$$

$FOC\left(K_{t+1}\right)$ may be rearranged to obtain:

$$\begin{aligned} \beta^t \lambda_t &= \beta^{t+1} \lambda_{t+1}\left(1+r_{t+1}\right) \\ \lambda_t &= \frac{\beta^{t+1}}{\beta^t} \lambda_{t+1}\left(1+r_{t+1}\right), \end{aligned} \tag{16}$$

which since $\beta=\frac{1}{(1+\rho)}$ gives

$$\lambda_t=\lambda_{t+1} \frac{\left(1+r_{t+1}\right)}{(1+\rho)}. \tag{17}$$

The FOCs for C_t and K_{t+1} combine to give the the intertemporal marginal utility relation known as the Keynes-Ramsey rule. We already know

$$\lambda_t=\frac{1}{C_t}=U'\left(C_t\right)$$

$$\lambda_{t+1}=\frac{1}{C_{t+1}}=U'\left(C_{t+1}\right)$$

$$\lambda_t=\lambda_{t+1} \frac{\left(1+r_{t+1}\right)}{(1+\rho)}.$$

Hence after substitution for the Lagrangians in (17) we have

$$\frac{1}{C_t}=\frac{1}{C_{t+1}} \frac{\left(1+r_{t+1}\right)}{(1+\rho)} \tag{18}$$

which in the present setup gives :

$$\frac{U'\left(C_t\right)}{U'\left(C_{t+1}\right)}=\frac{C_{t+1}}{C_t}=\frac{\left(1+r_{t+1}\right)}{(1+\rho)}. \tag{19}$$

Eq.(19) is one of the most important in all of macroeconomic theory, and we shall see it again in various forms. It implies that households' optimal consumption plan equates the marginal utility of an extra unit of consumption today $U'\left(C_t\right)$ to the discounted marginal utility of consuming next period $\left(U'\left(C_{t+1}\right) \cdot \frac{1}{(1+\rho)}\right)$ the marginal product $\left(1+r_{t+1}\right)$ of this consumption unit: $U'\left(C_t\right)=U'\left(C_{t+1}\right) \cdot \frac{\left(1+r_{t+1}\right)}{(1+\rho)}$.

Since $\ln\left(\frac{(1+r_{t+1})}{(1+\rho)}\right) = [\ln(1+r_{t+1}) - \ln(1+\rho)] \cong \left(r_{(t+1)} - \rho\right)$ when r and ρ are "small"[1], eq.(19) implies for all $t > 0$

$$\begin{aligned} \ln C_{t+1} - \ln C_t &= (r_{t+1} - \rho) \\ \ln C_{t+1}(1-L) &= (r_{t+1} - \rho). \end{aligned} \tag{20}$$

After multiplying the left- and right-sides of the second line of (20) by

$$\left(1 + L + L^2 + ... + L^t\right)$$

we obtain a solution for log consumption at $t+1$ in terms of the initial condition $\ln C_0$:

$$\begin{aligned} & \ln C_{t+1}(1-L) \cdot \left(1 + L + L^2 + ... + L^t\right) \\ = & \left[(r_{t+1} - \rho) \cdot \left(1 + L + L^2 + ... + L^t\right)\right] \end{aligned} \tag{21}$$

$$\begin{aligned} \Rightarrow & \ \ln C_{t+1}(1 - L^{t+1}) = \textstyle\sum_{j=0}^{t} L^j (r_{t+1} - \rho) \\ = & \ \textstyle\sum_{j=0}^{t} r_{t+1-j} - (t+1)\rho. \end{aligned}$$

If returns to saving are constant at, say, rate $\overline{r}$, (21) implies:

$$\ln C_{t+1} = \ln C_0 + (t+1) \cdot (\overline{r} - \rho). \tag{22}$$

The optimal consumption time path is therefore upward sloping from period $t = 0$ forward *when* the interest rate exceeds the rate of time preference, and conversely.

[1] Taylor's approximation is

$$\begin{aligned} \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \\ 0 &< x < 1. \end{aligned}$$

For small x the first term is very accurate.

Hence when $\overline{r} > \rho$ households behaving optimally will defer consumption today in favor of higher consumption later on. We shall see this famous equation in continuous time and in more general context when I revisit the Ramsey-Cass-Koopmans model during the growth theory lectures.

Returning to the household's program,

$$U(t=0) \Rightarrow Max_{\left\{C_t, N_t^s, K_{t+1}^s\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \ln C_t - \frac{(N_t^s)^{1+\phi}}{1+\phi} + \\ \lambda_t \cdot \left[r_t K_t^s + w_t N_t^s - \left(K_{t+1}^s - K_t^s\right) - C_t \right] \end{array} \right],$$

the FOC for choice of labor supply is:

$$FOC\left(N_t^s\right) \Rightarrow \frac{\partial U(t=0)}{\partial N_t^s} = \beta^t \left(-\left(N_t^s\right)^{\phi} + \lambda_t w_t \right) = 0 \tag{23}$$

which implies that

$$\begin{aligned} \left(N_t^s\right)^{\phi} &= \lambda_t w_t \\ U'\left(N_t^s\right) &= w_t U'\left(C_t\right). \end{aligned} \tag{24}$$

Hence labor is supplied to market activity such that the marginal (dis)utility of labor is equated to the wage *weighted by* the value to time t marginal utility of consumption opportunities that are afforded by the wage income obtained from an extra unit of labor supply. Put another way, at the optimum the marginal disutility of working must equal the marginal benefit of working, which is the value of the wage in units of marginal consumption.

We can now derive a dynamic labor supply equation. We know from (14) and (24) that

$$\left(N_t^s\right)^{\phi} = \frac{1}{C_t} w_t \tag{25}$$

and we know from the intertemporal symmetry of the sequential solution that

$$\left(N_{t+1}^s\right)^{\phi} = \frac{1}{C_{t+1}} w_{t+1}. \tag{26}$$

Hence

$$\frac{N^s_{t+1}}{N^s_t} = \left[\frac{w_{t+1}}{w_t}\left(\frac{C_{t+1}}{C_t}\right)^{-1}\right]^{\frac{1}{\phi}}. \tag{27}$$

Using the result in (19), $\frac{C_{t+1}}{C_t} = \frac{(1+r_{t+1})}{(1+\rho)}$, we find

$$\frac{N^s_{t+1}}{N^s_t} = \left[\frac{w_{t+1}}{w_t}\left(\frac{(1+r_{t+1})}{(1+\rho)}\right)^{-1}\right]^{\frac{1}{\phi}} \tag{28}$$

and taking logs we obtain

$$\left(\ln N^s_{t+1} - \ln N^s_t\right) = (1/\phi)\cdot\left[(\ln w_{t+1} - \ln w_t) - (r_{t+1} - \rho)\right] \tag{29}$$

where I again make use of the fact that $\ln(1+r_{t+1}) \cong r_{t+1}$, and $\ln(1+\rho) \cong \rho$.

An implication of (29) is that for given growth of wages, households choose a downward sloping time path for market labor supply when the interest rate exceeds the discount rate. In other words, when $r > \rho$ market work today is higher, and leisure is deferred.

Equation (29) implies

$$\ln N^s_{t+1}(1-L) = (1/\phi)(\ln w_{t+1} - \ln w_t) - (1/\phi)(r_{t+1} - \rho) \tag{30}$$

which after multiplication by $(1 + L + L^2 + ... + L^t)$ gives a solution for the level of log labor supply at period $t+1$ from the initial condition $\ln N^s_0$:

$$\ln N^s_{t+1}(1-L)\cdot\left(1 + L + L^2 + ... + L^t\right) = (1/\phi)\cdot\sum_{j=0}^{t} L^j \left[\begin{array}{c}(\ln w_{t+1} - \ln w_t)\\ -(r_{t+1} - \rho)\end{array}\right]$$

$$\ln N^s_{t+1}\left(1 - L^{t+1}\right) = (1/\phi)\cdot\sum_{j=0}^{t} L^j \left[\begin{array}{c}(\ln w_{t+1} - \ln w_t)\\ -(r_{t+1} - \rho)\end{array}\right]$$

$$\ln N^s_{t+1} = \ln N^s_0 + (1/\phi)\cdot\left[\begin{array}{c}(\ln w_{t+1} - \ln w_0)\\ -\left(\sum_{j=0}^{t} r_{t+1-j} - (t+1)\cdot\rho\right)\end{array}\right]. \tag{31}$$

For r_{t+1} equal to $\overline{r}$ at all t, the solution for log labor supply therefore is

$$\ln N_{t+1}^s = \ln N_0^s + (1/\phi) \cdot [(\ln w_{t+1} - \ln w_0) - (t+1) \cdot (\overline{r} - \rho)] \tag{32}$$

which, as already noted, means that for given wage changes the household's optimal labor supply is downward sloping over time (and, as shown earlier, optimal consumption is upward sloping) when the interest rate exceeds the rate of time preference. Hence, high real interest rates tend to produce a world of relatively low current consumption (high current saving and investment), and relatively high current propensity to supply market work.

5 Equilibrium Requirements and Outcomes

Given $K_{0,}$ a competitive equilibrium requires:

$Q_t = C_t + I_t$, the goods market clears
$N_t^s = N_t^d$, the labor market clears
$K_t^s = K_t^d$, the capital market clears
the sequences $\left\{K_t^d, N_t^d\right\}_{t=0}^{\infty}$ solve the producer's program
the sequences$\{C_t, N_t^s, K_t^s, I_t\}_{t=0}^{\infty}$ solve the consumer's program.

Using the firm's demand for labor relation in (8), which equates the wage to the marginal product of labor deployed, $w_t = (1-\alpha) A_t \left(K_t^d\right)^{\alpha} \left(N_t^d\right)^{-\alpha}$, in combination with the household's optimal labor supply relations in (24) – (26), $(N_t^s)^{\phi} = \lambda_t w_t = \frac{1}{C_t} w_{t,}$ and the requirement of competitive equilibrium, $N_t^s = N_t^d$, we can derive an equation for equilibrium employment:

$$\begin{aligned} \left(N_t = N_t^S\right) \;\; \Rightarrow \;\; N_t &= \left(\frac{1}{C_t} w_t\right)^{\frac{1}{\phi}} \quad \text{using } (24), (26) \\ &= C_t^{-\frac{1}{\phi}} \cdot \left[(1-\alpha) A_t (K_t)^{\alpha} (N_t)^{-\alpha}\right]^{\frac{1}{\phi}} \quad \text{using } (8) \end{aligned} \tag{33}$$

$$
\begin{aligned}
N_t \cdot N_t^{\frac{\alpha}{\phi}} &= C_t^{-\frac{1}{\phi}} \cdot \left[(1-\alpha) A_t (K_t)^{\alpha}\right]^{\frac{1}{\phi}} \\
&\implies (N_t)^{\frac{(\phi+\alpha)}{\phi}} = C_t^{-\frac{1}{\phi}} \cdot \left[(1-\alpha) A_t (K_t)^{\alpha}\right]^{\frac{1}{\phi}} \\
N_t &= \left\{ C_t^{-\frac{1}{\phi}} \left[(1-\alpha) A_t (K_t)^{\alpha}\right]^{\frac{1}{\phi}} \right\}^{\frac{\phi}{(\phi+\alpha)}} \\
N_t &= (1-\alpha)^{\frac{1}{(\phi+\alpha)}} A_t^{\frac{1}{(\phi+\alpha)}} C_t^{-\frac{1}{(\phi+\alpha)}} K_t^{\frac{\alpha}{(\phi+\alpha)}}. \qquad (34)
\end{aligned}
$$

For a given state of technology, A, the steady-states for the bare bones setup now follow directly. From the household's Euler equation in (19),

$$
\frac{C_{t+1}}{C_t} = \frac{(1+r_{t+1})}{(1+\rho)},
$$

we see that steady-state consumption, $C_{t+1} = C_t = C^*$ (or $\frac{C_{t+1}}{C_t} = 1$) requires $r^* = \rho$, where r^* is the equilibrium interest rate. From the capital accumulation equation in (2), it follows that at equilibrium conditional on A the capital stock satisfies $K_{t+1} = K_t = K^*|A$, and so investment, I^*, will be zero. (Recall there is no depreciation of the capital stock.) Hence equilibrium consumption equals equilibrium output (again conditional on a constant state of technology, A): $C^*|A = Q^*|A_t = A_t K^{*\alpha} N^{*^{1-\alpha}}$

To obtain *steady-state employment*, substitute the steady-state condition $C^* = Q^*$ into the employment equation in (34):

$$
\begin{aligned}
N^* &= (1-\alpha)^{\frac{1}{(\phi+\alpha)}} A^{\frac{1}{(\phi+\alpha)}} C^{*-\frac{1}{(\phi+\alpha)}} K^{*\frac{\alpha}{(\phi+\alpha)}} \\
N^* &= (1-\alpha)^{\frac{1}{(\phi+\alpha)}} A^{\frac{1}{(\phi+\alpha)}} \left(A K^{*\alpha} N^{*^{1-\alpha}}\right)^{-\frac{1}{(\phi+\alpha)}} K^{*\frac{\alpha}{(\phi+\alpha)}} \\
N^* \cdot (N^*)^{\frac{(1-\alpha)}{(\phi+\alpha)}} &= (1-\alpha)^{\frac{1}{(\phi+\alpha)}} \\
N^{*\frac{(\phi+1)}{\phi+\alpha}} &= (1-\alpha)^{\frac{1}{(\phi+\alpha)}} \\
N^* &= (1-\alpha)^{\frac{1}{(1+\phi)}}. \qquad (35)
\end{aligned}
$$

Notice that in general equilibrium steady-state employment (labor input to production)

(i) rises with labor's share of income, $\frac{Q'(N)\cdot N}{Q} = \frac{w\cdot N}{Q} = (1-\alpha)$:

$$\frac{dN^*}{d(1-\alpha)} \equiv \frac{d\left[(1-\alpha)^{\frac{1}{(1+\phi)}}\right]}{d(1-\alpha)} = \frac{1}{(1+\phi)} \cdot \left[(1-\alpha)^{\frac{1}{1+\phi}-1}\right] > 0$$

(ii) rises with the elasticity of the marginal value of time, ϕ :

$$\begin{aligned} \text{Since } \frac{d\ln N^*}{d\phi} &= \frac{1}{N^*}\cdot\frac{dN^*}{d\phi} \\ \frac{dN^*}{d\phi} &= \frac{d\ln N^*}{d\phi}\cdot N^* = \frac{d\left[\frac{1}{(1+\phi)}\ln(1-\alpha)\right]}{d\phi}\cdot(1-\alpha)^{\frac{1}{(1+\phi)}} \\ &= -\frac{\ln(1-\alpha)}{(1+\phi)^2}(1-\alpha)^{\frac{1}{1+\phi}} > 0 \end{aligned}$$

given $(0 < \alpha < 1)$ [2]

(iii) is not affected by the technology-productivity term A. This result follows from the specification of log utility; the income and substitution effects of rising A_t (rising productivity and real wages) cancel out.

The *steady-state stock of capital* may be obtained by using the equilibrium condition $r^* = \rho$ and substituting N^* into the demand for capital in (10):

$$r^* = \rho = \alpha A K^{*(\alpha-1)}(N^*)^{1-\alpha} \tag{36}$$

$$\rho = \alpha A K^{*(\alpha-1)}\left((1-\alpha)^{\frac{1}{(1+\phi)}}\right)^{1-\alpha} \tag{37}$$

which implies that the technology-conditional equilibrium steady-state capital

[2] If the above result seems counterintuitive, remember that ϕ is the elasticity of marginal utility of N^s, that is, the proportional change in marginal utility, $U'(N^s)$, generated by a proportional change in N^s, as shown earlier.

stock is

$$
\begin{aligned}
K^{*(1-\alpha)} \cdot \rho &= \alpha A \left((1-\alpha)^{\frac{1}{(1+\phi)}} \right)^{1-\alpha} \\
K^* &= \left(\frac{\alpha A}{\rho} \right)^{\frac{1}{(1-\alpha)}} (1-\alpha)^{\frac{1}{(1+\phi)}} \\
K^*|A &= \left(\frac{\alpha A (1-\alpha)^{\frac{(1-\alpha)}{(1+\phi)}}}{\rho} \right)^{\frac{1}{(1-\alpha)}} . \qquad (38)
\end{aligned}
$$

You should convince yourself that steady-state stock of capital

(i) rises with increases in capital's share, α

(ii) falls with increases in the rate of time preference, ρ

(iii) rises with increases in the elasticity of the marginal value of time, ϕ, and of course

(iv) increases with rising technological progress, A.

Finally, technology-conditional *equilibrium steady-state consumption and income* are:

$$
\begin{aligned}
C^*|A &= Q^*|A = AK^{*\alpha} N^{*^{1-\alpha}} \\
&= A \left(\frac{\alpha A (1-\alpha)^{\frac{(1-\alpha)}{(1+\phi)}}}{\rho} \right)^{\frac{\alpha}{(1-\alpha)}} (1-\alpha)^{\frac{(1-\alpha)}{(1+\phi)}} . \qquad (39)
\end{aligned}
$$

Comparative statics for C^*, Q^* follow from the results for and N^* and K^* in (35) and (38).

Self-Help Exercises:

In the Dynamic General Macro Equilibrium setup assume that the utility of consumption at each period takes the more general CIES form

$$
U(C_t) = \frac{C_t^{\left(1-\frac{1}{\sigma}\right)}}{\left(1-\frac{1}{\sigma}\right)}, \qquad \sigma > 0, \ \sigma \neq 1 \qquad (40)
$$

in place of the $\ln C_t$ specification of utility of consumption that was used in the lecture to define the household's program.

Using the utility of consumption function of eq.(40) derive and give economic interpretations of:

- the elasticity of marginal utility of consumption, as in eq.(5)
- the FOC for optimal consumption choices, as in eqs. $(13 - 14)$
- the Euler equation for optimal intertemporal consumption, as in eq.(19)
- an employment equation corresponding to eq.(35)

  Also:

- Show how the magnitude of $\sigma, \quad \sigma \neq 1, \quad \sigma \gtrless 1.0$, in eq.(40) affects the way equilibrium employment responds to the level of technological progress, A. In particular, discuss the economic intuition behind the response of N^* to A for $\sigma \gtrless 1.0$.