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Long-Term Wage Contracting and Monetary Policy Non-Neutrality

During our last meeting we saw how under rationality of expectations and fast ("new classical") market clearing adjustments of prices and wages, systematic demand management policy could not affect the real macroeconomy. In this lecture we shall see that a properly tuned aggregate demand policy can stabilize output fluctuations even with rational expectations, if some wages are predetermined vis-a-vis policy by long-term wage contracting institutions. The setup follows the seminal paper of Fischer (JPE, 1977). Throughout we assume that all agents have an information set at each decision period that includes outcomes up to and including that period. Generally wages for time t, $t+1, ...$are determined one period earlier, at time $t-1$. All variables are in logs.

1 Wage Formation

Multi-period contracts are observed in many economies, presumably because renegotiating wages at every period in spot-market fashion imposes significant transaction costs. Suppose that log nominal wages (net of productivity)[1], w, are set by overlapping, multi-period contracts, C, that specify wages one-period in advance for i periods covering $0 < f^{i\tau}_{t+\tau} < 1$ of all wages in the economy. For a given wage contract, wages can be indexed

$$w^i_{t+\tau}|C^i_t, \qquad \tau = 1, ...i. \tag{1}$$

The average (macroeconomic) log nominal wage at each time t, $\bar{w}_t$, is therefore the weighted sum of wages specified for time t by the overlapping, multi-period contracts

$$\bar{w}_t = \sum_{i=1}^{i.\max} \sum_{\tau=1}^{i} f^{i\tau}_t \; w^i_t|C^i_{t-\tau}. \tag{2}$$

As in Fischer (JPE, 1977), in order to illustrate the main points let us take $i.\max = 2$ and $f^{i\tau}_t = 1/2$, $\tau = 1, 2$. In this case

$$\bar{w}_t = \frac{1}{2}w_t|C^1_{t-1} + \frac{1}{2}w_t|C^2_{t-2}, \tag{3}$$

[1] w can then be thought of as what I denoted $\hat{w}$ in the aggregate supply lecture.

in other words, the observed average wage at t is the average of (i) the *first-period* log nominal wage from a two-period contract forged at time $t-1$ that specifies wages for periods t and $t+1$, and (ii) the *second-period* log nominal wage from a two-period contract forged at time $t-2$ that specifies wages for periods $t-1$ and t.

Suppose further that the objective served by wage contracts is to preserve the productivity-adjusted real wage. One can think of wage setters maximizing the function

$$C_t^i \Rightarrow \sum_{\tau=1}^{i} \delta^{\tau-1} \left(w_{t+\tau} - E_t p_{t+\tau}\right)^2 \tag{4}$$

with FOCs

$$\begin{aligned} 2\delta^{\tau-1} \left(w_{t+\tau} - E_t p_{t+\tau}\right) &= 0 && (5) \\ &\Rightarrow w_{t+\tau}^i | C_t^i = E_t p_{t+\tau}. && (6) \end{aligned}$$

For $i.\max = 2$ and $f_t^{i\tau} = 1/2$, the average log wage at time t is just

$$\bar{w}_t = \frac{1}{2} E_{t-1} p_t + \frac{1}{2} E_{t-2} p_t \tag{7}$$

with constituents that originate with the contractual wage specifications $w_t | C_{t-1}^1$ and $\frac{1}{2} w_t | C_{t-2}^2$, respectively.

2 Aggregate Supply and Demand

Aggregate supply and demand are nearly the same as in the Lucas setup we discussed during the last lecture. For simplicity and without loss of generality, we take the log natural output level to be zero, and set the scale parameter $\tilde{\beta} = 1$. However, shocks to supply and demand are now persistent (which is a reasonable characterization).

$$\begin{aligned} q_t^S &= (p_t - \bar{w}_t) + u_t^S && (8) \\ &= p_t - \frac{1}{2} \sum_{i=1}^{2} E_{t-i} p_t + u_t^S \\ u_t^S &= \rho_S u_{t-1}^S + \varepsilon_t, \qquad |\rho_S| < 1, \varepsilon_t \sim \left(0, \sigma_\varepsilon^2\right), white && (9) \end{aligned}$$

$$q_t^D = m_t - p_t - v_t^D \tag{10}$$

$$v_t^S = \rho_D v_{t-1}^D + \eta_t, \qquad |\rho_D| < 1, \eta_t \sim \left(0, \sigma_\eta^2\right), white. \tag{11}$$

$$Cov(\varepsilon, \eta) = 0. \tag{12}$$

Supply and demand shocks therefore exhibit stationary first-order autoregressive persistence, and have zero covariance.

3 Monetary Policy

Monetary policy is systematic, and it aims to stabilize the macroeconomy by accommodating supply shocks and offsetting demand shocks, which are observed with a one-period lag. The money supply is determined by[2]

$$m_t = \sum_{j=0}^{\infty} a_{j+1} u_{t-j-1}^S + \sum_{j=0}^{\infty} b_{j+1} v_{t-j-1}^D. \tag{13}$$

Since monetary policy reacts to lagged shocks, m_t is fully anticipated by all agents at time $t-1$:

$$E_{t-1}(m_t) = m_t. \tag{14}$$

4 Equilibrium Price

Markets are competitive, and so we find price by equating demand and supply.

$$\begin{aligned} q_t^D &= q_t^S \qquad (15) \\ m_t - p_t - v_t^D &= p_t - \frac{1}{2}\sum_{i=1}^{2} E_{t-i} p_t + u_t^S \end{aligned}$$

$$\Rightarrow p_t = \frac{1}{2} m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-i} p_t - \frac{1}{2}\left(v_t^D + u_t^S\right). \tag{16}$$

All agents know that the price level will satisfy eq. (16).

Now keep in mind the law of iterated expectations

$$\begin{aligned} E_{t-j}E_{t-i} &= E_{t-j}, \qquad j > i \qquad (17) \\ &= E_{t-i} \qquad j < i \\ &\Rightarrow E_{t-2}E_{t-1} = E_{t-2} \\ &\Rightarrow E_{t-1}E_{t-2} = E_{t-2}. \end{aligned}$$

[2] As in Fischer (1977), demand shocks are written with negative sign in the equation for q^D and, therefore, the m policy reaction function offsets v_{t-j}. It would be more intuitive to sign v postively in q^D (velocity shocks) and negatively in the reaction function, but to avoid confusion I follow Fischer's convention.

Let us now find the terms $\sum_{i=1}^{2} E_{t-i}p_t$ in the intermediate solution for p_t in (16), just as agents in the economy could.

Begin with $E_{t-2}p_t$:

$$\begin{aligned} E_{t-2}p_t &= E_{t-2}\left[\frac{1}{2}m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-i}p_t - \frac{1}{2}\left(v_t^D + u_t^S\right)\right] && (18) \\ E_{t-2}p_t &= \frac{1}{2}E_{t-2}m_t + \frac{1}{4}E_{t-2}E_{t-1}p_t + \frac{1}{4}E_{t-2}E_{t-2}p_t \\ &\quad -\frac{1}{2}E_{t-2}\left(v_t^D + u_t^S\right) \\ E_{t-2}p_t &= \frac{1}{2}E_{t-2}m_t + \frac{1}{4}E_{t-2}p_t + \frac{1}{4}E_{t-2}p_t - \frac{1}{2}E_{t-2}\left(v_t^D + u_t^S\right) \end{aligned}$$

$$\Rightarrow \quad E_{t-2}p_t = E_{t-2}m_t - E_{t-2}\left(v_t^D + u_t^S\right). \qquad (19)$$

Next consider $E_{t-1}p_t$:

$$\begin{aligned} E_{t-1}p_t &= E_{t-1}\left[\frac{1}{2}m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-i}p_t - \frac{1}{2}\left(v_t^D + u_t^S\right)\right] && (20) \\ E_{t-1}p_t &= \frac{1}{2}E_{t-1}m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-1}E_{t-1}p_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-1}E_{t-2}p_t \\ &\quad -\frac{1}{2}E_{t-1}\left(v_t^D + u_t^S\right) \\ E_{t-1}p_t &= \frac{1}{2}m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-1}p_t + \frac{1}{4}E_{t-2}p_t - \frac{1}{2}E_{t-1}\left(v_t^D + u_t^S\right). \end{aligned}$$

Now substitute in for $E_{t-2}p_t$, which is known to agents one period later, at period $t-1$:

$$E_{t-1}p_t = \frac{1}{2}m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-1}p_t + \frac{1}{4}\left[E_{t-2}m_t - E_{t-2}\left(v_t^D + u_t^S\right)\right] - \frac{1}{2}E_{t-1}\left(v_t^D + u_t^S\right) \quad (21)$$

$$\Rightarrow \quad E_{t-1}p_t = \frac{2}{3}m_t + \frac{1}{3}E_{t-2}m_t - \frac{2}{3}E_{t-1}\left(v_t^D + u_t^S\right) - \frac{1}{3}E_{t-2}\left(v_t^D + u_t^S\right). \quad (22)$$

5 Equilibrium Output

Having solved for p, we can find q from q^S or q^D using (16), (19) and (21):

$$q_t = p_t - \left[\frac{1}{2}E_{t-1}p_t + \frac{1}{2}E_{t-2}p_t\right] + u_t^S \quad (23)$$

$$= \left\{\frac{1}{2}m_t + \frac{1}{4}\sum_{i=1}^{2} E_{t-i}p_t - \frac{1}{2}\left(v_t^D + u_t^S\right)\right\} - \left\{\left[\frac{1}{2}\sum_{i=1}^{2} E_{t-i}p_t\right] + u_t^S\right\}$$

$$= \left\{\frac{1}{2}m_t - \frac{1}{2}\left(v_t^D + u_t^S\right)\right\} - \left\{\left[\frac{1}{4}\sum_{i=1}^{2} E_{t-i}p_t\right] + u_t^S\right\}$$

$$= \frac{1}{2}m_t - \frac{1}{2}\left(v_t^D + u_t^S\right) - \frac{1}{4}E_{t-1}\left[\frac{2}{3}m_t + \frac{1}{3}E_{t-2}m_t - \frac{2}{3}E_{t-1}\left(v_t^D + u_t^S\right) - \frac{1}{3}E_{t-2}\left(v_t^D + u_t^S\right)\right] - \frac{1}{4}E_{t-2}\left[E_{t-2}m_t - E_{t-2}\left(v_t^D + u_t^S\right)\right] + u_t^S$$

$$= m_t \cdot \left(\frac{1}{2} - \frac{1}{4}\cdot\frac{2}{3}\right) - E_{t-2}m_t \cdot \left(\frac{1}{4}\cdot\frac{1}{3} + \frac{1}{4}\right) - E_{t-1}\left(v_t^D + u_t^S\right)\cdot\left(\frac{2}{3}\right) - E_{t-2}\left(v_t^D + u_t^S\right)\cdot\left(\frac{1}{4}\cdot\frac{1}{3} + \frac{1}{4}\right) - \left(v_t^D + u_t^S\right)\cdot\frac{1}{2} + u_t^S$$

$$q_t = \frac{1}{3}(m_t - E_{t-2}m_t) + \frac{1}{2}\left(u_t^S - v_t^D\right) + \frac{1}{6}E_{t-1}\left(v_t^D + u_t^S\right) + \frac{1}{3}E_{t-2}\left(v_t^D + u_t^S\right). \quad (24)$$

What can we say about the term $(m_t - E_{t-2}m_t)$? The monetary policy rule in eq.(13), along with the first-order autoregressive persistence of shocks, imply

$$\begin{aligned} m_t &= \frac{a_1 u_{t-1}^S}{(1-L)} + \frac{b_1 v_{t-1}^S}{(1-L)} \quad (25) \\ &= m_{t-1} + a_1 u_{t-1}^S + b_1 v_{t-1}^S \\ &= m_{t-1} + a_1 \rho_S u_{t-2}^S + a_1 \varepsilon_{t-1} + b_1 \rho_D v_{t-2}^D + b_1 \eta_{t-1}. \end{aligned}$$

Hence

$$\begin{aligned} E_{t-2}m_t &= E_{t-2}(m_{t-1}) + E_{t-2}\left[a_1 \rho_S u_{t-2}^S + a_1 \varepsilon_{t-1} + b_1 \rho_D v_{t-2}^D + b_1 \eta_{t-1}\right] \quad (26) \\ &= m_{t-1} + a_1 \rho_S u_{t-2}^S + b_1 \rho_D v_{t-2}^D. \end{aligned}$$

Therefore

$$(m_t - E_{t-2}m_t) = a_1 \varepsilon_{t-1} + b_1 \eta_{t-1}. \quad (27)$$

Note that ε_{t-1} and η_{t-1} are supply and demand shocks, respectively, which the monetary authority reacts to when setting m_t, but which cannot have affected the setting of w_t in contracts forged at time $t-2$ (the second period of the two period contract making up 1/2 of the time t average wage $\bar{w}_t$). This reaction advantage of the monetary authority gives it scope to stabilize aggregate output because half of the wage structure is determined by "old" wage contracts.

Note that the AR1 structure of demand and supply shocks implies that

$$v_t^D = \rho_D^2 v_{t-2}^D + \rho_D \eta_{t-1} + \eta_t \quad (28)$$

$$u_t^S = \rho_S^2 u_{t-2}^S + \rho_S \varepsilon_{t-1} + \varepsilon_{t.} \quad (29)$$

Thus output may be written

$$\begin{aligned} q_t &= \frac{1}{3}(m_t - E_{t-2}m_t) \\ &-\frac{1}{2}\left(v_t^D - u_t^S\right) \\ &+\frac{1}{6}E_{t-1}\left(v_t^D + u_t^S\right) \\ &+\frac{1}{3}E_{t-2}\left(v_t^D + u_t^S\right) \end{aligned} \tag{30}$$

$$\begin{aligned} q_t &= \frac{1}{3}\left(a_1\varepsilon_{t-1} + b_1\eta_{t-1}\right) \\ &-\frac{1}{2}\left(\rho_D^2 v_{t-2}^D + \rho_D\eta_{t-1} + \eta_t - \rho_S^2 u_{t-2}^S + \rho_D\varepsilon_{t-1} + \varepsilon_t\right) \\ &+\frac{1}{6}\left(\rho_D^2 v_{t-2}^D + \rho_D\eta_{t-1} + \rho_S^2 u_{t-2}^S + \rho_D\varepsilon_{t-1}\right) \\ &+\frac{1}{3}\left(\rho_D^2 v_{t-2}^D + \rho_S^2 u_{t-2}^S\right) \end{aligned}$$

$$\begin{aligned} q_t &= \frac{1}{2}(\varepsilon_t - \eta_t) \qquad \leftarrow \text{cannot be responded to by } m_t \text{ policy} \\ &+\frac{1}{3}\left[\varepsilon_{t-1}\cdot(a_1 + 2\rho_S) + \eta_{t-1}\cdot(b_1 - \rho_D)\right] \\ &+\rho_S^2 u_{t-2}^S. \qquad \leftarrow \text{cannot be responded to by } m_t \text{ policy} \\ &\qquad\qquad \text{in a way that affects real wages} \end{aligned} \tag{31}$$

6 Volatility of log Output Under Active and Passive Policy

We now want to find the variance of log output in (31), noting that given the autoregressive persistence

$$\begin{aligned} u_t^S &= \rho_S u_{t-1}^S + \varepsilon_t \\ \sigma_u^2 &= \rho_S^2\sigma_u^2 + \sigma_\varepsilon^2 \\ &\Rightarrow \ \sigma_u^2 = \frac{\sigma_\varepsilon^2}{(1-\rho_S^2)}. \end{aligned}$$

Taking the variance q gives

$$\sigma_q^2 = \frac{1}{4} \cdot \left(\sigma_\varepsilon^2 + \sigma_\eta^2\right) + \frac{1}{9} \cdot \left[\sigma_\varepsilon^2 \cdot (a_1 + 2\rho_D)^2 + \sigma_\eta^2 \cdot (b_1 - \rho_S)^2\right] + \rho_S^4 \sigma_u^2. \tag{32}$$

Next, collect terms, remembering that $\sigma_u^2 = \frac{\sigma_\varepsilon^2}{(1-\rho_S^2)}$:

$$\begin{aligned} \sigma_q^2 &= \sigma_\varepsilon^2 \cdot \left[\frac{1}{4} + \frac{1}{9} \cdot (a_1 + 2\rho_S)^2 + \frac{\rho_S^4}{(1-\rho_S^2)}\right] \\ &\quad + \sigma_\eta^2 \cdot \left[\frac{1}{4} + \frac{1}{9} \cdot (b_1 - \rho_D)^2\right] \end{aligned} \tag{33}$$

How would a monetary authority following the policy rule

$$\begin{aligned} m_t &= \sum_{j=0}^{\infty} a_{j+1} u_{t-j-1}^S + \sum_{j=0}^{\infty} b_{j+1} v_{t-j-1}^D \\ &= m_{t-1} + a_1 u_{t-1}^S + b_1 v_{t-1}^S \end{aligned}$$

set the reaction coefficients a_1 and b_1 in order to stabilize output fluctuations, that is, in order to minimize σ_q^2 in eq.(33)? The optimal choices obviously are

$$a_1 = -2\rho_S \tag{34}$$

$$b_1 = \rho_D \tag{35}$$

which gives output variance as

$$\begin{aligned} \sigma_q^2 &= \sigma_\varepsilon^2 \cdot \left[\frac{1}{4} + \frac{1}{9} \cdot (-2\rho_S + 2\rho_S)^2 + \frac{\rho_S^4}{(1-\rho_S^2)}\right] \\ &\quad + \sigma_\eta^2 \cdot \left[\frac{1}{4} + \frac{1}{9} \cdot (\rho_D - \rho_D)^2\right] \end{aligned}$$

$$\sigma_q^2 = \sigma_\varepsilon^2 \cdot \left[\frac{1}{4} + \frac{\rho_S^4}{(1-\rho_S^2)}\right] + \frac{1}{4}\sigma_\eta^2. \tag{36}$$

The optimal money supply rule is therefore

$$m_t = m_{t-1} - 2\rho_S u_{t-1}^S + \rho_D v_{t-1}^S. \tag{37}$$

Given the specifications of supply (eq.8) and demand (eq.10), optimal policy changes m each period in order to accommodate unfavorable supply shocks (negative realizations of u_{t-1}^S that lower q^S and raise p) and to offset unfavorable demand shocks (negative realizations of v_{t-1}^S that raise q^D and p).

Note that the policy non-neutrality, as pointed out before, is achieved by affecting the real wage of workers in the second period of a two-period contract. Unlike the monetary authority which sets m every period, the time t wages specified by "old" contracts cannot take account of the (price) effects of the unanticipated components of $u^S_{t-1}$and v^S_{t-1}(ε_{t-1} and η_{t-1}, respectively).

Finally, if one were to take a model like that outlined above as a serious approximation to a macroeconomy, simple experiments might be performed to calibrate the reduction in volatility of output produced by a well tuned activist monetary policy. For example, comparing the variance of log output generated by activist monetary policy – eq.(36) – with a passive policy – $a_1 = b_1 = 0$, giving $m_t = m_{t-1}$ – one would find a reduction in σ^2_q of around 20 percent for plausible settings of the persistence parameters and relative variances of supply and demand shocks.